5 Work, Energy and Power

Energy used by a body for existence is known as metabolic energy to perform activity and sustain life. Food consumed by a human body is broken into chemical energy in form of ATP and heat. This energy is converted to work, thermal energy and stored fat. This energy is essential for the functioning of the brain and manual labour. Hence, this chapter is important to learn the elements of energy in the physical world.

Topic Notes

- *Work Done and Gravitational Potential Energy*
- *Power and Conservation of Linear Momentum*

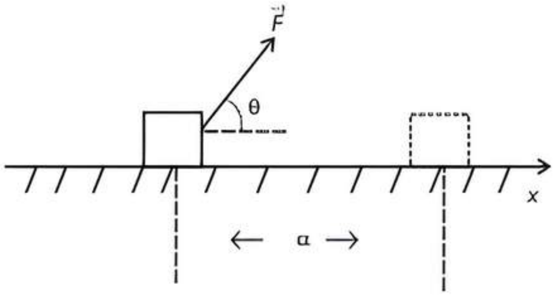
WORK DONE AND GRAVITATIONAL POTENTIAL ENERGY

1

TOPIC 1

WORK

In physics, work stands for 'mechanical work'. It is said to be done by force when the body is displaced through the same distance in the direction of the applied force. However, while there may be no displacement in the direction of applied force, no work is stated to be carried out *i.e.*, work done is zero, when the displacement of the body in the direction of the force is zero.



If θ is the angle, which $\vec{F}$ makes with +ve x-direction of the displacement, then the component of $\vec{F}$ in the direction of displacement is F *cos* θ.

$$\therefore \quad W = (F\cos\theta)d \qquad \text{—(i)}$$

If displacement is in the direction of the force applied, θ= 0° from equation (i),

$$W = (F\cos 0^\circ)\, d = F.d$$

An equation can be written as,

$$W = F.d \qquad \text{—(ii)}$$

Thus, work done by a force is the dot product of force and displacement.

In terms of rectangular component $\vec{F}$ and $\vec{d}$, may be written as:

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

and

$$\hat{d} = x\hat{i} + y\hat{j} + z\hat{k}$$

From equation (ii)

$$W = xF_x + yF_y + zF_z$$

∴ Work is a scalar quantity, *i.e.*, it has magnitude only and no direction.

Dimensions and Units of work

$$\text{Work} = \text{Force} \times \text{Distance}$$

$$W = [M^1 L^1 T^{-2}] \times [L]$$

(Dimensional formula)

$$W = [ML^2T^{-2}]$$

There are two different units of work:

(1) Absolute Unit:

(i) **Joule:** It is an absolute unit of S.I. system.

From, $W = Fd\cos\theta$

$1 \text{ joule} = 1 \text{ Newton} \times 1 \text{ metre} \times \cos 0^\circ = 1\text{N–m}$

(ii) **Erg:** It is an absolute unit of work of CGS system.

From, $W = F.d\cos\theta$

$$1 \text{ erg} = 1 \text{ dyne} \times 1\text{cm} \times \cos 0^\circ$$
$$= 1 \text{ dyne-cm}$$

(2) Gravitational units: (also known as a practical unit of work)

(i) **Kilogram-meter (kg-m)**- It is said to be a 1 kg-m work, when a force of 1 kg required to move a body through a distance of 1 m in the direction of applied force.

$$W = Fd\cos\theta$$
$$1 \text{ kg/m} = 1 \text{ kgf} \times 1\text{m} \times \cos 0^\circ$$
$$= 9.8 \text{ N} \times 1\text{m} = 9.8 \text{ joule}$$
$$1 \text{ kg/m} = 9.8 \text{ J}$$

(ii) **Gram-centimetre (g-cm)**- It is said to be g-cm, work when a force of 1 gm required to moves a body through a distance of 1 cm in the direction of applied force.

$$W = Fs\cos\theta$$
$$1 \text{ g/cm} = 1 \text{ gf} \times 1 \text{ cm} \times \cos 0^\circ$$
$$1 \text{ g/cm} = 980 \text{ dyne} \times 1 \text{ cm} \times 1$$
$$1 \text{ g/cm} = 980 \text{ erg}$$

Important

➥ *There are three types of work done:*

(1) *Positive work.* $W = \vec{F}.\vec{d} = F.d\cos\theta$

∴ when θ is acute (< 90°) cosθ is +ve.

Hence, work done is +ve.

(2) *Negative work. As* $W = \vec{F}.\vec{d} = Fd\cos\theta$

∴ when θ is obtuse (> 90°) cosθ is -ve.

Hence, work done is -ve.

(3) *Zero work, when a force applied or the displacement or both are zero.*

$$W = Fd \cos\theta - 0 \text{ (zero)}$$

Again, when the angle θ *between* $\vec{F}$ *and* $\vec{d}$ *is* 90°

$$\cos\theta = \cos 90^\circ = 0$$

∴ *Hence, work done is zero.*

Example 1.1: The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative:

(A) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket,

(B) Work done by gravitational force in the above case,

(C) Work done by friction on a body sliding down an inclined plane,

(D) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,

(E) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

[NCERT]

Ans: Work done, W = F.S = Fs $\cos\theta$

(A) Work done 'positive', because force is acting in the direction of displacement *i.e.*, $\theta = 0^\circ$.

(B) Work done is negative, because force is acting against the displacement *i.e.*, $\theta = 180^\circ$.

(C) Work done is negative, because force of friction is acting against the displacement *i.e.*, $\theta = 180^\circ$.

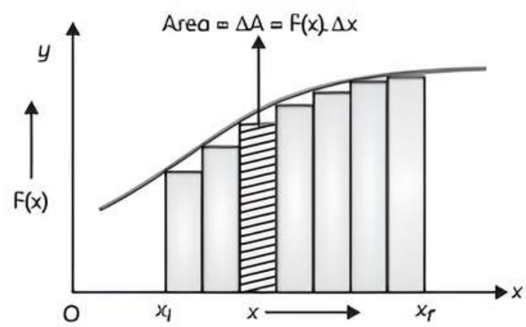
(D) Work done is positive, because body moves in the direction of applied force *i.e.*, $\theta = 0^\circ$.

(E) Work done is negative, because the resistive force of air opposes the motion *i.e.*, $\theta = 180^\circ$.

TOPIC 2

WORK DONE BY A VARIABLE FORCE

Let us consider a varying force in one dimension.

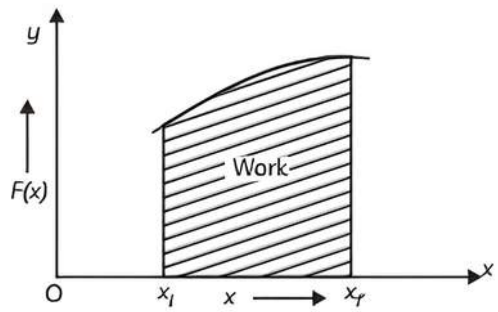


If displacement Δx is small, force F(x) can be regarded as approximately constant and the work done:

$$\Delta W = F(x).\ \Delta x$$

Total work done, $W = \int_{x_i}^{x_f} F(x)dx$

If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit, but the sum approaches a definite value equal to the area under the curve.



$$\sum_{x_i}^{x_f} (x)\Delta x = \int_{x_i}^{x_f} (x)dx$$

Thus, for varying force the work can be expressed as a definite integral of force over displacement.

Example 1.2: An object moves along *x*-axis from *x* = 0 to *x* = 4 m under the influence of force given by F = (5 – 3*x* + 2x^2) N. Calculate work done by the force.

Ans. Work done by a force F to displace an object through a distance *dx* is given by

$$dW = Fdx$$

∴ Total work done to displace an object from *x* = 0 to *x* = 4 m can be calculated as:

$$\int dW = \int_{x=0}^{x=4} Fdx$$

$$= \int_0^4 (5 - 3x + 2x^2)dx$$

$$= 5[x]_0^4 - 3\left[\frac{x^2}{2}\right]_0^4 + 2\left[\frac{x^3}{3}\right]_0^4$$

$$= 5[4-0] - 3\left(\frac{16}{2} - 0\right) + 2\left(\frac{64}{3} - 0\right)$$

$$= 38.67 \text{ J}$$

TOPIC 3

ENERGY

The energy of a body is defined as its capacity for doing work.

Since, the energy of a body is equal to the total quantity of work done. Hence, it is a scalar quantity. The dimensional formula of energy is $[ML^2T^{-2}]$ *i.e.*, same as that of work. Energy is measured in the same units as the work. The S.I. unit of energy is joule and the CGS unit is erg.

Kinetic Energy

Energy possessed by a body by the virtue of its motion is called Kinetic energy.

$$KE = \frac{1}{2}mv^2$$

Relation between Kinetic Energy and Linear Momentum

As we already know,

$$p = mv \qquad \text{...(i)}$$

Where m is the mass and v is the velocity of the body.

And,

$$KE = \frac{1}{2}mv^2$$

From eqn. (i), we get

$$KE = \frac{p^2}{2m}$$

$$p = \sqrt{2mKE}$$

Work Energy Theorem

This theorem states that the work done by a force on a body is equal to the change in kinetic energy of the body.

Consider a mass m moving with a velocity u. Let a force F be applied on the body. So that, it is accelerated with an acceleration 'a'.

Then $\qquad F = ma$

If s be the distance travelled by the body arriving in its accelerated motion, then the work done by the force F is given by,

$$W = Fs = mas \quad (\because F = ma) \text{ ...(i)}$$

Let the body acquires velocity v after travelling a distance s, then from

$$v^2 - u^2 = 2as,$$

We have,

$$W = m\left(\frac{v^2 - u^2}{2s}\right) \times s$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Where, $\frac{1}{2}mv^2$ = final kinetic energy

$\frac{1}{2}mu^2$ = initial kinetic energy

The difference between the final and initial kinetic energies is the change in kinetic energy of the body, (ΔK.E.). where, Δ means change.

$\because$ W = Change in K.E. of a body = Δ (K.E.)

This is known as the work-energy theorem.

Work done on the body in order to increase its velocity from u to v, is given by,

$$W = \int_u^v mvdv = m\int_u^v vdv = m\left[\frac{v^2}{2}\right]_u^v$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

This result is the mathematical form of the work-energy theorem which states that work done by a force acting on a body is equal to the change produced in the K.E. of the body.

If work done on the body in order to increase its velocity from zero to v.

$$W = \int_0^v mvdu = m\int_0^v vdv = m\left[\frac{v^2}{2}\right]_0^v$$

$$W = \frac{1}{2}mv^2$$

Example 1.3: A bullet of mass 20 g strikes a target with a velocity of 160 ms^{-1} and is brought to rest after piercing 10 cm into it. Calculates the average force of resistance offered by the target.

Ans. Let F be the average resistance force offered by the target. Then according to work-energy theorem;

$$W = -Fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Here, $\quad s = 10 \text{ cm} = 0.1 \text{ m}$

$m = 20 \text{ g} = 0.02 \text{ kg}$

$u = 160 \text{ m/s}, v = 0$

$$\therefore \quad -F \times 0.1 = \frac{1}{2} \text{m} \times 0.02 \times 0^2 - \frac{1}{2} \times 0.02 \times 160^2$$

$$F = \frac{0.02 \times 160^2}{2 \times 0.1} = 2560 \text{ N}$$

Work-Energy Theorem for Variable Force

We know that, $K.E = \frac{1}{2}mv^2$

On differentiating both sides w.r.t 't', we get,

$$\frac{d(K.E)}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

$$= \frac{1}{2}m \times 2v\frac{dv}{dt}$$

$$= mv\frac{dv}{dt}$$

But $\frac{dv}{dt} = a$

$\therefore$ $\frac{d(K.E)}{dt} = mav$

Or $\frac{d(K.E)}{dt} = Fv$ $\quad (\because F = ma)$

Since $v = \frac{dx}{dt}$

$\therefore$ $\frac{d(K.E)}{dt} = F\frac{dx}{dt}$

or $d(K.E) = Fdx$ ...(i)

When $x = x_i$

$K.E = (K.E)_f$

Integrating equation (i) between these limits,

$$\int_{(K.E)_i}^{(K.E)_f} d(K.E) = \int_{x_i}^{x_f} Fdx$$

or $\int_{x_i}^{x_f} Fdx = [K.E]_{(K.E)_i}^{(K.E)_f}$

$$= (K.E)_f - (K.E)_i = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

But $\int_{x_i}^{x_f} Fdx$ = work done (W) by the variable force

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \text{change in K.E.}$$

Work done on a body by a variable force F in displacing the body from initial position x_i to the final position x_f is given by:

$$W = \int_{x_i}^{x_f} F(x)\,dx \quad \text{...(ii)}$$

Let the force F be acting on the body of mass m such that the body is accelerated along x-axis with an acceleration a_x.

$$F(x) = ma_x \qquad \left(\because \frac{dx}{dt} = v_x\right)$$

Hence, equation (i) becomes

$$W = \int_{u_i}^{u_f} F(x)dx = ma_x\,dx$$

Now, $a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\cdot\frac{dx}{dt}$ ...(iii)

$$= v_x\frac{dv_x}{dx} \qquad \left(\because \frac{dx}{dt} = v_x\right)$$

Let v_1 be the velocity of the body at the position x_i and v_2 be the velocity of the body at position x_f. Now, put the value of equation (iii) in equation (i) charge the limits of integration from x_i to x_f to v_1 to v_2.

$$W = \int_{x_i}^{x_f} mv_x\frac{dv_x}{dx}dx = \int_{v_1}^{v_2} mv_x dv_x$$

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \text{change in K.E.}$$

Work-energy theorem holds in all inertial frames. It can be extended to non-inertial frames, provided we include the pseudo force in the calculation of the net force acting on the body under consideration.

When K.E. increases, the work done is positive and when K.E. decreases, the work done is negative.

Example 1.4: A toy rocket of mass 0.1 kg has a small fuel of mass 0.02 kg, which it burns out in 8 s. Starting from rest on a horizontal smooth track, it gets a speed of 20 m/s after the fuel is burnt out. What is the approximate thrust of the rocket? What is the energy content per unit mass of the fuel? (Ignore the small mass variation of the rocket during fuel burning).

Ans. Here, $m = 0.1$ kg, $u = 0$, $v = 20$ m/s, $t = 3$ s

The thrust of the rocket

$$= ma = m\frac{v-u}{t}$$

$$= 0.1 \times \frac{20-0}{3} = \frac{2}{3}\text{ N}$$

The kinetic energy gained by the rocket

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.1 \times (20)^2 = 20\text{ J}$$

Energy content per unit mass of the fuel

$$= \frac{\text{Total energy}}{\text{Mass of the fuel}} = \frac{20\text{J}}{0.02\text{kg}}$$

$$= 1000\text{ Jkg}^{-1}$$

Potential Energy

The potential energy of a body is defined as the energy possessed by the body by virtue of its position or configuration in the same field. Thus, the potential energy that can be associated with the configuration (or arrangement) of a system of objects that extend forces on one another. Obviously, if the configuration of the system changes, then its potential energy changes.

Gravitational Potential Energy

A system of two materials which exerts a gravitational attractive force on each other possesses potential energy.

The gravitational potential energy of a body is the energy which is possessed by the body by virtue of its position above the surface of the earth.

We can calculate the gravitational potential energy by given formula.

Gravitational potential energy, $g(h) = mgh$

Where, m = mass of a body

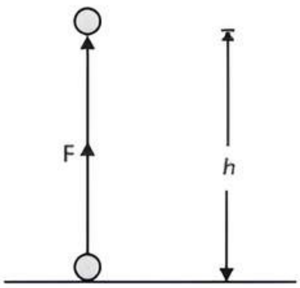
g = acceleration due to gravity on the surface of Earth

h = height through which the body is raise

If we assume that height h is not too large and the value of g is practically constant over this height, then the force applied just to overcome gravitation attraction is,

$$F = mg$$

As the distance moved is in the direction of the forces applied:



Work done = Force × Distance

$$W = F \times h = mgh$$

Note that, we have taken the upward direction to be positive.

∴ Work done by applied force = $+ mgh$

However, work done by gravitational force = $- mgh$

This work gets stored as potential energy. The gravitational potential energy of a body as a function of height (h) is denoted by $g(h)$ and it is negative of work done by the gravitational force in raising the body to that height.

Gravitational Potential Energy = $g(h) = mgh$

Important

➥ *It is easily seen that the gravitational force F equals the negative of the derivative of g(h)= –mgh*

➥ *The –ve sign indicates that the gravitational force is downward. When released, the ball comes down with an increasing speed. Just before it hits the ground, its speed is given by the kinematic relation.*

$$v^2 = 2gh$$

The equation can be written as:

$$\frac{1}{2} mv^2 = mgh$$

which shows that the gravitational potential energy of the object at height h, when the object is released, manifests itself as kinetic energy of the object on reaching the ground.

Example 1.5: Case Based:

Hydropower plants are places where the generation of electricity takes place with the help of water. When the moving water, possessing some kinetic energy, hits the turbine present in the dam, the kinetic energy of the water gets converted into mechanical energy moves the turbines and then, ultimately, it leads to the production of electrical energy.

(A) Hydroelectric generating station is supplied from a reservoir of a capacity 5×10^6 cubic meter at a head of 200 meters. Find the total energy available in kwh, if the overall efficiency is 75%.

(B) A hydroelectric reservoir can supply water continuously at a rate of 100 m^3/s. The head is 75 m. The theoretical power that can be developed is (MW) is:

(a) 73.5 (b) 65.7
(c) 68.5 (d) 70.8

(C) In hydroelectric power plants, ... reduces the water hammer effect in the pen stock.

(a) Spillway (b) Trash rack
(c) Surge tank (d) forebay

(D) Assertion (A): Initial expenses for the establishment of a hydroelectric power station are high.

Reason (R): Hydroelectricity is an inexhaustible resource.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false

(E) The operating cost of the hydroelectric power station is low is an advantage of hydroelectric power station over the thermal power station. Give reason.

Ans. (A) A hydroelectric generating station is a plant that produces electricity by using water to plant the turbine, which in turn drives the generator.

Potential energy, $E = mgh$

Total output energy, E' = Potential energy × Overall efficiency

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Where, M = mass

g = gravitational acceleration

h = height

Given that overall efficiency = 75%

Reservoir capacity,

$(V) = 5 \times 10^6\ m^3$

Water head,

$h_0 = 200$ m

Density of water

$= 997\ kg/m^3$

$g = 9.8\ m/s^2$

P.E. can be calculated as:

$$E = (n \times V) \times 9.8 \times 200$$
$$= 997 \times 5 \times 10^6 \times 9.8 \times 200$$
$$= 9.77 \times 10^{12}\ J$$
$$= 2.71 \times 10^6\ kwh$$

Hence, total output energy can be calculated as:

$$E' = 2.71 \times 10^6 \times 0.75$$
$$= 2 \times 10^6\ kwh$$

(B) *(a) 73.5*

Explanation: $Q = 100\ m^3/s$

$h = 75$ m

$P = Qgh$

$= 1000 \times 100 \times 9.81 \times 75$

$= 73.5 \times 10^6$

$= 73.5$ MW

(C) *(c) Surge tank*

Explanation:

(1) The excess water from the dam is discharged through the spillway at a permissible level. It is also known as the safety value of a dam.

(2) It is used in hydroelectric power plants to filter the water before it flows towards the turbine.

(3) Surge tables are usually provided in high or medium-head plants when there is a considerable distance between the water source & the power unit, necessitating a long pen stock.

(4) It is an enlarged body of water at the nitrate (dam) to store more quantity of water.

(D) *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: Hydroelectric electricity is produced with shifting water. Due to the fact, the source of hydroelectric power is water, hydroelectric power plants are generally positioned on or armour of a water source. Hydropower is a sort of renewable power, and as soon as the power plant is built, it produces little to no waste.

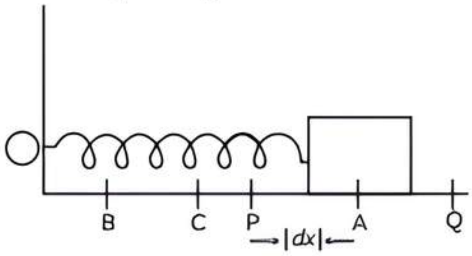
(E) It requires no fuel, as water is used for the generation of electrical energy. It is quite neat and clean, as no smoke or ash is produced. It requires very small running charges because water is the source of energy which is available at free of cost.

Potential Energy of a Spring

Potential energy of a spring is the energy associated with the state of compression or expression of an elastic spring.

To calculate it, consider an elastic spring OA of negligible mass. The end O of the spring is fixed to a right support, and a body of mass m is attached to the free end A.

Let the spring be oriented along x-axis and the body of mass m lie on a perfectly frictionless horizontal table.



The position of the object A, when spring is unstretched, is chosen as the origin.

When the spring is compressed or elongated, it tends to recover to its original length, on an account of elasticity. The force trying to bring the spring back to its original configuration is called restoring force or spring force.

Let us calculate the potential energy of the mass M, when then spring is pulled from the mean position C up to a point P, such that

$$CP = x.$$

As the spring gets stretched, the restoring force F is set up in the spring due to elasticity.

According to Hooke's law;

$$F \propto x$$
$$F = -kx$$

Where, the constant of proportionality k is called force constant of the spring or simply spring constant.

The –ve sign shows that the restoring force acts in a direction opposite to the direction in which the displacement increases.

Suppose that, the block is further displaced through an infinitesimally small distance PA = dx. The restoring force can be supposed to remain practically uncharged, as the increase in length is infinitesimally small. The small work done in increasing the length of the spring by dx

$$dW = Fdx = kxdx$$

The work done in increasing the length of the spring by an amount x can be calculated by integrating the above between limits, $x = 0$ to $x = k$ i.e.,

$$W = \int_0^x kx dx = k\int_0^x x dx = k\left|\frac{x^2}{2}\right|_0^x = k\left|\frac{x^2}{2} - 0\right|$$

Or $\quad W = \frac{1}{2} kx^2 \quad$ –(i)

This work done is stored in the system as its potential energy at point P i.e., when displacement of the block from equilibrium position is x Potential Energy. of the system. When the block is pulled up to the point Q, can be obtained by setting $x = r$ in equation (i).

Thus,

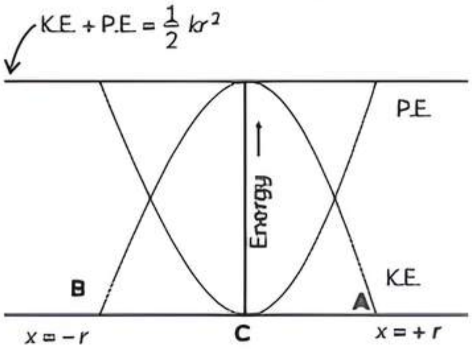
P.E. of the system at point Q $= \frac{1}{2} k(r)^2 = \frac{1}{2} kr^2$

At point Q the block is at rest.

Hence, K.E. of the system at point Q = 0.

At A.C, $\quad x = 0$.

$\therefore$ P.E of the system at point C $= \frac{1}{2} k(0)^2 = 0$



The loss in potential energy, when the system returns to the position C, appears as an equal increase in kinetic energy. Therefore,

K.E. of the system at point C $= \frac{1}{2} kr^2 - 0 = \frac{1}{2} kr^2$

P.E. of the system at point B can be found by setting $x = r$ in the equation (i)

$\therefore$ P.E. of the system at point B $= \frac{1}{2} k(-r)^2 - \frac{1}{2} kr^2$

Obviously, the K.E. of the system at point B = 0

As said above,

P.E. of the system at point P $= \frac{1}{2} kx^2$

The decrease in P.E. when the system returns point Q to P appears as the equal increase in kinetic energy.

$\therefore$ K.E. of the system at Point P $= \frac{1}{2} kr^2 - \frac{1}{2} kx^2$

If we plot P.E and K.E. against the displacement x, the graph will be as depicted by two dotted curves;

The sum of K.E. and P.E. is shown by the thick horizontal straight line and is always equal to $\frac{1}{2} kr^2$.

Important

➥ *The P.E. of the spring is at maximum, since the spring is stretched at its maximum extension. The P.E. due to gravity is at minimum, since the mass is lifted the least amount above ground.*

Example 1.6: **Point out the correct alternative:**

(A) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

(B) Work done by a body against friction always results in a loss of its kinetic/potential energy.

(C) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces of the system.

(D) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies. [NCERT]

Ans: **(A)** Potential energy of the body decreases because the body in this case goes closer to the Centre of the force.

(B) Kinetic energy, because friction does its work against the motion.

(C) Internal forces cannot change the total or net momentum of a system. Hence the rate of change of total momentum of many particle system is proportional to the external force on the system.

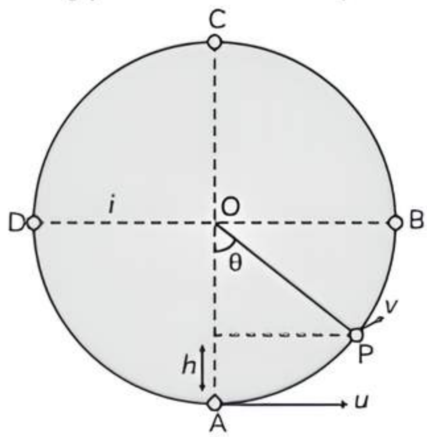
(D) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/ total energy of the system of two bodies.

Motion in a Vertical Circle

Vertical circular motion (VCM) is a form of circular motion that occurs when the body moves in a vertical plane.

In the example at hand, the velocity of the body varied at different points. It displays non-uniform circular motion. The earth's gravity is acting on the body in this motion.

Velocity at any point on vertical loop:



If u is the initial velocity imparted to the body at its lowest point, then the velocity of the body at height h is given by

$$v = \sqrt{u^2 - 2gh}$$
$$= \sqrt{u^2 - 2gl(1-\cos\theta)}$$

Here, l is the string's length.

Tension at any point on vertical loop:

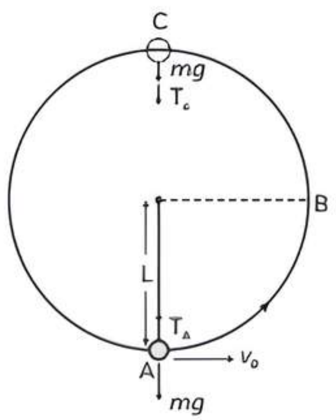
Tension at general point P,

$$T = mg\cos\theta + \frac{mv^2}{l}$$

Table : Various conditions for vertical:

Velocity at lowest point	Condition
$u_A > \sqrt{5gl}$	Tension in the string will not be zero at any of the point and body will continue the circular motion.
$u_A = \sqrt{5gl}$	Tension at highest point C will be zero and body will just complete the circle.
$\sqrt{2gl} < u_A < \sqrt{5gl}$	Particle will not follow circular motion. Tension in string become zero somewhere between points B and C whereas velocity remain positive. Particle leaves circular path and follow parabolic trajectory.
$u_A = \sqrt{2gl}$	Both velocity and tension in the string becomes zero between A and B and particle will oscillate along semi-circular path.
$u_A < \sqrt{2gl}$	Velocity of particle becomes zero between A and B but tension will not be zero and the particle will oscillate about the point A.

Example 1.7: A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity v_0 at the lowest point A such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, C. This is shown in Fig.



Obtain an expression for:

(A) v_0

(B) the speeds at points B and C

(C) the ratio of the kinetic energies $\left(\frac{K_B}{K_C}\right)$ at B and C.

Comment on the nature of the trajectory of the bob after it reaches the point C.

Ans. (A) Let the particle at the top most point be u m/s

As the particle is in vertical circular motion;

So, the centripetal acceleration is being provided by mg alone as string has slacked so,

$$mg = \frac{mu^2}{L}$$

So, $u = \sqrt{gL}$ at the top most point (when string slacks)........Speed of C

Now appliying the energy conservation between A and C

We have,

$$\frac{mv_0^2}{2} + 0 = \frac{mu^2}{2} + mg(2L)$$

so, $$\frac{mv_0^2}{2} = \frac{mu^2}{0} + mg(2L)$$

so, $$v_0 = \sqrt{5gL} \text{ ....Speed of A}$$

(B) For point C

Applying conservation of energy

$$\frac{mv_0^2}{2} + 0 = \frac{mu_B^2}{2} + mg(L)$$

$$\frac{mv_0^2}{2} = \frac{mu_B^2}{2} + mg(L)$$

so, $$u_B = \sqrt{3gL} \text{ ....Speed of B}$$

(C) Ratio of kinetic energy will be

$$\frac{KE_B}{KE_C} = \frac{\frac{mu_B^2}{2}}{\frac{mu^2}{2}}$$

$$= \frac{u_B^2}{u^2} = \frac{2gL}{gL} = \frac{3}{1}$$

The object will complete the circular path and its trajectory will be circular as string has slacked at the highest point.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. According to the work-energy theorem, the work done by the net force on a particle is equal to the change in its:
(a) kinetic energy
(b) angular momentum
(c) linear momentum
(d) potential energy [Diksha]

Ans. *(a) kinetic energy*

Explanation: Kinetic energy is the energy of motion. Observable as the movement of an object, particle or set of particles. The quantity of rotation of a body, which is the product of its moment of inertia and its angular velocity. The momentum of translation being a vector quantity in classical physics equal to the product of the mass and the velocity of the center of mass. Potential energy is the energy that is stored in an object due to its position relative to some zero position.

2. A man holds a bucket by applying 10 N force. Then he moves forward with a distance of 5 m and climbs up a vertical distance of 10 m. The total work done by him is:
(a) 100 J (b) 150 J
(c) 50 J (d) 200 J
[NCERT Exemplar]

Ans. *(a) 100 J*

Explanation: $s = 5$ m
$F = 10$ N
$\theta = 90°$

Work done is given by,
$$W_1 = Fs\cos\theta$$
$$W_1 = 10 \times 5 \times \cos 90° = 0$$

Here, $F = 10$ N
$s = 10$ m
$\theta = 0°$

So, work done, $W_2 = Fs\cos\theta$
$= 10 \times 10 \times \cos 0° = 100$ J

$\therefore$ Total work done $= W_1 + W_2 = 0 + 100$ J
$= 100$ J

Related Theory

➥ *If the angle is 180°, then the work done is known as negative work done. Negative work removes or dissipates energy from the system.*

3. According to work-energy theorem, the work done by the net force on a particle is equal to the change in its:
(a) linear acceleration
(b) Potential energy
(c) linear momentum
(d) Kinetic energy [Delhi Gov. SQP 2022]

Ans. *(a) linear acceleration*

Explanation: The principle of work and Kinetic energy also known as the work-energy theorem states that the work done by the sum of all forces acting on a particle equals the change in the Kinetic energy of the particle.

4. Calculate the average force necessary to stop a bullet of mass 20 g and speed 250 m/s as it penetrates a wooden block to a distance of 12 cm.
(a) 6×10^3 N (b) 5.2×10^3 N
(c) 5.4×10^2 N (d) 6.2×10^2 N
[Diksha]

Ans. *(b) 5.2×10^3 N*

Explanation: If force F Newton be the retarding force, then the work done by force is given by,
$$W = F \times s = F \times 0.12 \text{ J}$$
Loss of kinetic energy
$$= \frac{1}{2} \times \frac{20}{1000} \times 250 \times 250 \text{ J}$$
$$= 625 \text{ J}$$
(This K.E. is consumed in stopping the bullet and is converted into heat energy)
Apply work-energy theorem,
$$F \times 0.12 = 625$$
$$F = \frac{625}{0.12} = 5.2 \times 10^3 \text{ N}$$
Note that, the retarding force is nearly 30, 000 times the weight of the bullet.

⚠ Caution

➥ *If work done by the two bodies in same time then,*
$$Power \propto \frac{1}{time}$$
i.e., the body which performs given work in less time posses more power and vice versa.

5. A bullet of mass 20 g is found to pass two points 30 m apart in a time interval of 4 s. Calculate the kinetic energy of the bullet if it moves with constant speed.
(a) 0.5625 J (b) 0.4625 J
(c) 0.3267 J (d) 0.2345 J
[NCERT Exemplar]

Ans. *(a) 0.5625 J*

Explanation: Speed,

$$v = \frac{\text{Distance}}{\text{time}} = \frac{30\text{ m}}{4\text{ s}}$$

$$= 7.5\text{ m/s}$$

$$\text{K.E.} = \frac{1}{2} \times 0.02 \times (7.5)^2$$

$$= 0.5625\text{ J}$$

Related Theory

➥ *It is observed that, $KE \propto m$ and $KE \propto v^2$. Thus, a heavier body and a body moving faster possess greater energy and vice-versa.*

6. A ball is thrown vertically up with a velocity of 20 m/s. At what height, will its K.E. be half of its original value?

(a) 9.25 m **(b) 7.89 m**

(c) 10.20 m **(d) 11.22 m**

Ans. *(c) 10.20 m*

Explanation: Work done against gravitation = Change in K.E.

$$mgh = \frac{1}{2}mv^2 - \frac{1}{2}\times\frac{1}{2}mv^2 = \frac{1}{4}mv^2$$

$$h = \frac{v^2}{4g} = \frac{20\times20}{4\times9.8} = 10.20\text{ m}$$

Caution

➥ *Students should know that it is assumed that over the distance h, a constant force mg is required to move the body up. Obviously, h should not be large. Otherwise, the value of g may change.*

7. A particle is projected at an angle of 60 degree to the horizontal with a kinetic energy E. The K.E at the highest point is:

(a) $\frac{E}{2}$ **(b) $\frac{E}{4}$**

(c) $\frac{E}{3}$ **(d) 2E**

[Delhi Gov. SQP 2022]

Ans. *(b) $\frac{E}{4}$*

Explanation: Let initial velocity of the particle be *v*.

$$\therefore \quad \text{K.E.} = \frac{1}{2}mv^2$$

at the highest point of its flight

$$v = u\cos60° = \frac{u}{2}$$

∴ Kinetic energy at this point

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{mu^2}{4} = \frac{\text{K.E.}}{4}$$

8. A spring of force constant 5×10^3 Nm^{-1} is stretched initially by 5 cm from the unstretched position. Then, the work required to stretch it further by another 5 cm is:

(a) 12.50 Nm **(b) 18.75 Nm**

(c) 25.00 Nm **(d) 6.25 Nm**

Ans. *(b) 18.75 Nm*

Explanation: Work done in increasing the extension from initial extension x_i to final extension x_f is given by

$$W = \frac{1}{2}k(x_f^2 - x_i^2)$$

Here, $k = 5 \times 10^3$ Nm^{-1}, x_i = 5 cm = 0.05 m

$$x_f = 5 + 5 = 10\text{ cm} = 0.1\text{ m}$$

$$W = \frac{1}{2} \times 5 \times 10^3 \times (0.1^2 - 0.05^2)$$

$$= 18.75\text{ Nm}$$

Caution

➥ *Students must know that a stout spring has a large value of force constant, while for a delicate spring, the value of spring constant is low.*

9. Find the potential energy stored in a ball of 5 kg placed on the ground.

(a) 121.20 J **(b) 147.15 J**

(c) 227.31 J **(d) 182.21 J**

Ans. *(b) 147.15 J*

Explanation: Given, m = 5 kg, h = 3 m, g = 9.81 m/s^2

We know,

Potential energy = *mgh*

= 5 × 3 × 9.81

= 147.15 J

Related Theory

➥ *The potential energy of a body which is subjected to a conservative force is uncertain upto a certain limit. This is because the point of zero potential energy is a matter of choice. For the spring potential energy, the zero potential energy is the equilibrium position of the oscillating mass.*

10. Find the amount of work done by gravitational force when an object of mass 10 kg is moved by 10 m horizontally.

(a) 1000 J **(b) 100 J**

(c) 10 J **(d) Zero** **[Diksha]**

Ans. *(d) Zero*

Explanation: The angle between the force and displacement will be 0, because body will move horizontally. So, work done by the gravitational force is zero.

Assertion-Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these question from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

11. Assertion (A): According to law of conservation of mechanical energy, change in potential energy is equal and opposite to the change in kinetic energy.

Reason (R): Mechanical energy is not a conserved quantity.

[Delhi Gov. QB 2022]

Ans. (c) *A is true but R is false.*

Explanation: Because mechanical energy is conserved in a process, the change in potential energy equals and opposes the change in kinetic energy.

12. Assertion (A): A person working on a horizontal road with a load on this head does not work.

Reason (R): No work is said to be done if the direction of force and displacement of load are perpendicular to each other.

Ans. (a) *Both A and R are true and R is the correct explanation of A.*

Explanation: The work done

$$W = \vec{F}.\vec{S} \cos\theta$$

When a person walks on a horizontal road with load on his head.

Then $\theta = 90°$.

Hence, $W = Fs \cos 90° = 0$

Thus, no work is done by a person.

13. Assertion (A): A spring has potential energy, both when it is compressed or stretched.

Reason (R): In compressing or stretching, work is done in the spring against the restoring force.

Ans. (a) *Both A and R are true and R is the correct explanation of A.*

Explanation: The work done on the spring against the restoring force is stored as potential energy in both conditions when it is compressed or stretched.

14. Assertion (A): Water at the foot of the waterfall is always at different temperatures from that at the top.

Reason (R): The potential energy of water at the top is converted into heat energy during falling.

[Diksha]

Ans. (a) *Both A and R are true and R is the correct explanation of A.*

Explanation: When the water is at the top of the fall it has potential energy *mgh* (where *m* is the mass of the water and *h* is the height of the fall). On falling, this potential energy is converted into kinetic energy which further converts into heat energy and so the temperature of water increases.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

15. *In a giant vacuum chamber heavy and light objects are dropped at the same time to see which will hit the ground faster. Though some objects, like feathers, seem to fall slower because of air resistance. The bowling ball and feathers are released, falling gracefully to the ground with neither accelerating farther than the other. The objects both stay in unison as they descend at the exact same time. Original experiments made by Galileo and Sir Isaac Newton lead to the theory of gravitation, though it also became the foundation of Albert Einstein's theory of relativity.*

(A) A light and a heavy body have equal momenta. Which one has greater kinetic energy?

(B) What is the final velocity of the ball if it is dropped from a certain height and takes 10 s to reach the ground. The air resistance is not taken into account? [take g = 10 m/s^2]

(C) The potential energy of a freely falling object decreases progressively. Does this violate the law of conservation? Why?

Ans. (A) Since, momenta of heavy body and light body are same, thus velocity of light body is greater than that of heavy body. Also, kinetic energy is proportional to the square of velocity. Hence, lighter body has greater kinetic energy.

(B) Applying first equation of motion under free fall.

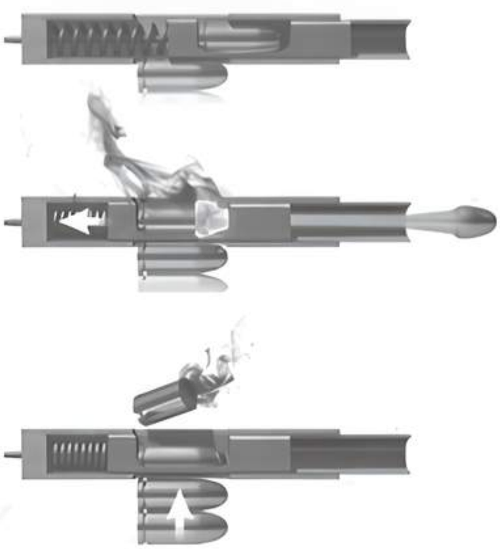
$$u = 0 \text{ m/s}$$
$$a = g = 10 \text{ m/s}^2$$
$$t = 10 \text{ s}$$

As we know that:

$$v = u + at$$
$$v = gt$$
$$v = 10 \times 10$$
$$v = 100 \text{ m/s}$$

(C) No, the process does not violate the law of conservation of energy because when the body falls from a height, its potential energy changes into kinetic energy progressively. A decrease in the potential energy is equal to an increase in the kinetic energy of the body. During the process, the total mechanical energy of the body remains conserved.

16. *Kinetic energy depends upon the frame of reference and is always positive. A bullet fired from a gun has very high kinetic energy and so, it can easily penetrate any object. This is because of the large amount of velocity possessed by the bullet. Although the mass of the bullet is less, its high speed renders double the amount of kinetic energy.*



(A) A bullet of mass 50 g is moving with a velocity of 500 ms^{-1}. It penetrates 10 cm into a still target and comes to rest. Kinetic energy possessed by the bullet is:

(a) 6250 joule (b) 7000 joule
(c) 6500 joule (d) 7250 joule

(B) Using the values in the above question, the average retarding force offered by the target will be:

(a) 60250 N (b) 62500 N
(c) 60000 N (d) 70000 N

(C) A bullet of mass 10 g moving with a velocity of 100 m/s hits a wooden log and penetrates it up to a thickness of 5 cm. The resistance force of log is:

(a) 200 N (b) 500 N
(c) 1000 N (d) 600 N

[NCERT Exemplar]

(D) A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest, assuming that it faces constant resistance to motion?

(a) 0.5 cm (b) 10 cm
(c) 5 cm (d) 1 cm [Diksha]

(E) A bullet of mass 10 g is fired horizontally with a velocity 1000 ms^{-1} from a rifle situated at a height 50 m above the ground. If the bullet reaches the ground with a velocity 500 m/s, the work done against air resistance (magnitude) in the trajectory of the bullet in Joule (g = 10 m/s^2) is:

(a) 5005 J (b) 3755 J
(c) 3750 J (d) 17.5 J

Ans. (A) *(a) 6250 joule*

Explanation: $\text{K.E.} = \frac{1}{2} \times \frac{50}{1000} \times (500)^2$

$$= \frac{25000}{4} \text{ Joule}$$

$$= 6250 \text{ Joul}$$

(B) *(b) 62500 N*

Explanation: $v^2 = u^2 + 2as$

$$0 = (500)^2 + 2a\frac{10}{100}$$

$$a = -12{,}50000 \text{ m/s}^2$$

Average retarding force

$$F_{av} = ma$$

$$= \frac{50}{1000} \times 12{,}50000$$

$$F_{av} = 62500 \text{ N}$$

(C) *(c) 1000 N*

Explanation: Since finally, the bullet stops. Therefore, loss in kinetic energy

Initial kinetic energy = Final kinetic energy

Let F be the average resisting force.

$$m = 10 \text{ g or } 0.01 \text{ kg}$$

$$s = 5 \text{ cm} = 0.05 \text{ m}$$

So, $$\frac{mv^2}{2} = Fs$$

$$F \times 0.05 = \frac{0.01 \times 100^2}{2} = 1000 \text{ N}$$

(D) *(d) 1 cm*

Explanation: Given that: $s = 3$ cm

$$2as = \left[\frac{u}{2}\right]^2 - u^2$$

$$2a.3 = \left[\frac{u}{2}\right]^2 - u^2$$

$$\Rightarrow \quad \frac{-3u^2}{4} = 2a3$$

$$a = \frac{-u^2}{8}$$

$$0 - \left[\frac{u}{2}\right]^2 = 2a.x$$

$$\frac{-u^2}{4} = 2\left[\frac{-u^2}{8}\right] \times x$$

$$x = 1 \text{ cm}$$

(E) *(b) 3755 J*

Explanation: Work done against air resistance is the energy tends to.

Given that:

$$m = 10 \text{ g}$$

$$u = 1000 \text{ m/s}$$

$$v = 500 \text{ m/s}$$

$$g = 10 \text{ m/s}^2$$

which is $W = (mgh + \frac{1}{2} mu^2) - \frac{1}{2} mv^2$

$$= \frac{1}{2} m (u^2 - v^2) + mgh$$

$$= 10^{-2} (\frac{1}{2} (1000^2 - 500^2) + 10 \times 50)$$

$$= 3755 \text{ J}$$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

17. **Spring A and B are identical except that A is stiffer than B, *i.e.*, force constant $k_A > k_B$. In which spring is more work expended if they are stretched by the same amount?** **[Delhi Gov. QB 2022]**

Ans. The force applied to both springs is the same for A, the spring constant is more than that of B, but the expansion is smaller, therefore the work for A is likewise less than that of B.

$$W = \frac{1}{2}Kx^2$$

$$\frac{W_A}{W_B} = \frac{K_A}{K_B}, \text{ for same } x$$

As $K_A > K_B$ so, $W_A > W_B$.

18. **A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by;**

$$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k} \text{ N}$$

where, $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along *x*, *y* and *z*-axis of the system respectively. What is the work done by this force in moving the body from a distance of 4 m along the *z*-axis? **[NCERT Exemplar]**

Ans. Since the body is displaced 4 m along *z*-axis only,

$$\therefore \quad \vec{s} = 0\hat{i} + 0\hat{j} + 4\hat{k}$$

$$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$$

Work done,

$$\vec{W} = \vec{F}.\vec{s}$$

$$= (-\hat{i} + 2\hat{j} + 3(\hat{k})(0\hat{i} + 0\hat{j} + 4\hat{k})$$

$$= 12 \ (\hat{k}.\hat{k}) \text{ Joule} = 12 \text{ Joule}$$

19. **A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$, where $a = 5\text{m}^{1/2}\text{s}^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2$ m?** **[Diksha]**

Ans. Here, $m = 0.5$ kg

$u = a\,x^{3/2}$, $a = 5\ m^{1/2}\ s^{-1}$

Initial velocity at $x = 0$, $v_1 = ax = 0$

Final velocity at $x = 2$, $v_2 = a\,(2)^{3/2} = 5 \times (2)^{3/2}$

Work done = Increase in K.E.

$$= \frac{1}{2} m (v_2^2 - v_1^2)$$

$$= \frac{1}{2} \times 0.5[5 \times (2)^{3/2})^2 - 0] = 50 \text{ J}$$

20. A person trying to lose weight (dieters) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each. Assume that the potential energy lost each time she lowers the mass is dissipated, find out how much work she does against the gravitational force? [Diksha]

Ans. Here, $m = 10$ kg

$h = 0.5$ m, $n = 1000$

Work done against gravitational force,

$$w = (mgh)n$$
$$= 1000(10 \times 9.8 \times 0.5)$$
$$= 49000 \text{ J}$$

21. A body is attached to the lower end of a vertical spiral spring and it is gradually lowered to its equilibrium position. This stretches the spring by a length *x*. If the same body attached to the same spring is allowed to fall suddenly. What would be the mass stretching in this case? [Diksha]

Ans. When spring is gradually lowered to its equilibrium position,

$$kx = mg$$

$$\therefore \quad x = \frac{mg}{k}$$

When spring is allowed to fall suddenly, it oscillates about its mean position.

Let y is the amplitude of vibration then at lower extreme, by the conservation of energy,

$$\Rightarrow \quad \frac{1}{2}ky^2 = mgy$$

$$y = 2\frac{2mg}{k} = 2x$$

22. Give an example in which a force does work on a body but fails to change its K.E. [Delhi Gov. QB 2022]

Ans. When a body is dragged at a steady velocity on a rough, horizontal surface. The body is being worked on, but K.E. stays unaltered.

23. A policeman fires a bullet of mass 50 gram with a speed of 200 m/s in a wooden piece of thickness 2.0 cm. The bullet leaves the wooden piece with only 10% of its initial kinetic energy. By how much percentage has the speed of the bullet been reduced?

Ans. The initial kinetic energy of the bullet is

$$\text{K.E.} = \frac{1}{2}mv_1^2$$

$$= \frac{1}{2} \times (50 \times 10^{-3} \text{ kg}) \times (200 \text{ m/s})^2$$

$$= 1000 \text{ J}$$

Final K.E. is only 10% of the initial energy, that is

$$\frac{1}{2}mv_1^2 = 100 \text{ J}$$

$$v_f = \sqrt{\frac{2 \times 100\text{J}}{(50 \times 10^{-3})\text{kg}}} \text{ m/s}$$

Reduction in speed is

$$= 200 \text{ m/s} - 63.2 \text{ m/s}$$
$$= 136.8 \text{ m/s}$$

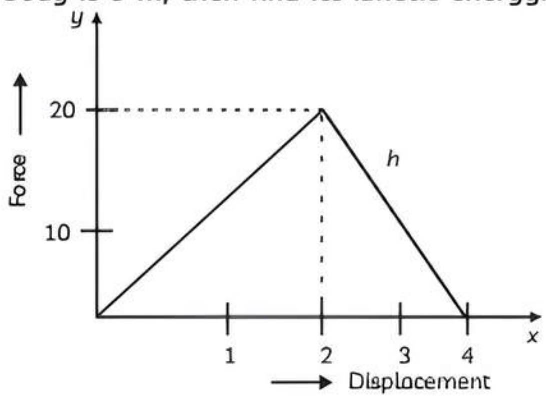
Percentage reduction is,

$$= \frac{136.8 \text{ m/s}}{200 \text{ m/s}} \times 100$$
$$= 68.4\%$$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

24. The graph between the resistive force *F* acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 kg and the initial velocity is 2 m/s. When the distance covered by the body is 5 m, then find its kinetic energy.



[Diksha]

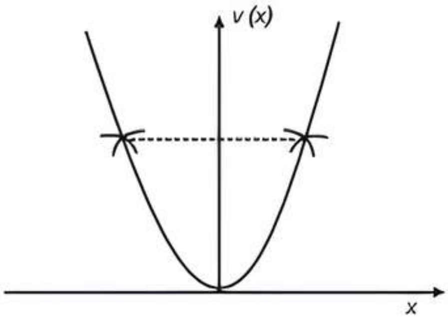
Ans. Initial kinetic energy of the body

$$\frac{1}{2}mu^2 = \frac{1}{2} \times 25 \times 2^2$$
$$= 50 \text{ Joule}$$

Final kinetic energy = Initial energy - work done against resistive force (Area between graph and displacement axis)

$$= 50 - \frac{1}{2} \times 4 \times 20$$
$$= 50 - 40 = 10 \text{ J}$$

25. The potential energy function for a particle executive linear simple harmonic motion is given by, $v = 9x - \frac{kx^2}{2}$, where k is the force constant of the oscillator. For $k = 0.5\ Nm^{-1}$, the graph of $v(x)$ versus x is shown in figure.



Show that a particle of total energy 15 J moving under this particle turn back when it reaches, $x = \pm 2$ m. [NCERT Exemplar]

Ans. Here force constant $k = 0.5\ Nm^{-1}$ and total energy of particle E = 1 J. The particle can go up to a maximum distance x_m, where its total energy is transformed into elastic potential energy

$$E = \frac{1}{2}kx_m^2$$

$$x_m = \sqrt{\frac{2E}{k}}$$

$$= \sqrt{\frac{2\times 1}{0.5}}$$

$$= \sqrt{4} = \pm 2\,m$$

26. Choose the correct alternative:

(A) If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy.

(B) The energy required to launch an orbiting satellite out of Earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of Earth's influence. [Delhi Gov. SQP 2022]

Ans. (A) Kinetic energy; A spacecraft revolving in its orbit has zero potential energy. A system's kinetic energy (+ve) and potential energy is added to determine its overall energy, since the earth satellite system is bound system. The overall energy of the satellite is negative. As a result, the satellite's energy is the inverse of its kinetic energy.

(B) Less; A stationary object at the same height has less energy than an orbiting satellite. The orbit contributes this extra energy. Compared to a stationary item, it needs less energy to move it away from the influence of the earth.

27. A 3.0 kg mass of body initially at rest which is move forward in horizontal direction of force 0.60 N on a smooth table. Find the work done 9.0 J the force in 9.05 s and show that this equals the change in the kinetic energy of the body.

Ans. The acceleration produced in the body by the horizontal force

$$a = \frac{F}{m}$$

$$= \frac{0.60\ N}{3.0\ kg}$$

$$= 0.2\ ms^{-2}$$

The distance moved in the horizontal direction by the body in t = 9.0 sec is,

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2}(0.2)(9.0)^2$$

$$s = 8.1\ m$$

The work done by the force in 9.0 sec is,

$$W = Fs$$

$$= 0.60\ N \times 8.1\ m$$

$$= 4.86\ J$$

Let v be the speed of the body after 9.0 s, then,

$$v = u + at = 0 + (0.2)(9.0)$$

$$= 1.8\ ms^{-1}$$

The kinetic energy of the body after 9.0 s is

$$K.E. = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(1.8)^2(3)$$

$$= 4.86\ J$$

The initial kinetic energy was zero.

$\therefore$ Change in kinetic energy is $\Delta K.E. = 4.86$ J

This is the same as the work done (work-energy theorem)

28. The length of a steel wire increases by 0.5 cm when it is loaded, with a weight of 5 kg. Calculate:

(A) Force constant of the wire.

(B) Work done in stretching the wire.

[Diksha]

Ans. (A) If a force F applied to a wire increases its length by x, then accordingly to Hooke's law,

$$F = kx$$

Where, k is force constant.

Given,

$$F = mg = 5 \times 10 = 50\ N$$

$$x = 0.5\ cm = 0.5 \times 10^{-2}\ m$$

$$k = \frac{F}{x} = \frac{50}{0.5\times 10^{-2}}$$

$= 1.0 \times 10^4 \text{ Nm}^{-1}$

(B) Work done in stretching the wire,

$$W = \frac{1}{2} kx^2$$

$$= \frac{1}{2} \times 1.0 \times 10^4 \times (0.5 \times 10^{-2})^2$$

$$W = 0.125 \text{ J}$$

29. A bullet is moving with the speed of 500 m/s just pierces a wooden block 5 cm thick. What will be the speed required to just pierce a block 8 cm thick?

Ans. The K.E. of the bullet is just spent in doing work against the resistive force offered by the block. Thus, if v_1 and v_2 be the speeds of the bullet of mass m (say) required to just cover distances (thickness) s_1 and s_2 in the block against the resistive force F then,

$$\frac{1}{2}mv_1^2 = Fs_1 \quad \text{...(i)}$$

and $$\frac{1}{2}mv_2^2 = Fs_2 \quad \text{...(ii)}$$

From eqn. (i) and (ii), we have,

$$\frac{v_1}{v_2} = \sqrt{\frac{s_1}{s_2}}$$

Here, $v_1 = 500$ m/s, $s_1 = 5$ cm, $s_2 = 8$ cm, $v_2 = ?$

$$\therefore \quad v_2 = v_1\sqrt{\frac{s_2}{s_1}}$$

$$v_2 = 500 \times \sqrt{\frac{8}{5}}$$

$$v_2 = 500 \times \sqrt{1.6}$$

$$v_2 = 500 \times 1.26$$

$$v_2 = 630 \text{ ms}^{-1}$$

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

30. A raindrop of mass 2 g is falling from a height of 1.00 km. It is the ground with a speed of 60.0 ms^{-1}. Find the work done:

(A) by the gravitational force

(B) by the unknown resistive force

($g = 10$ m/s^2)

Ans. (A) Assuming that the drop is initially at rest, the change in the kinetic energy of the drop is:

$$\Delta K = \frac{1}{2} mv^2 = 0$$

$$= \frac{1}{2} \times (2.00 \times 10^{-3} \text{ kg}) \times (60.0 \text{ ms}^{-1})^2$$

$$= 3.6 \text{ kgm}^2\text{s}^{-2} = 3.6 \text{ J}$$

Work done by the gravitational force is,

$$W_g = mgh$$

$$= (2.00 \times 10^{-3} \text{ kg}) \times (10 \text{ m/s}^2) \times (100 \times 10^5 \text{ m})$$

$$= 20 \text{ J}$$

(B) From the work-energy theorem, we have

$$\Delta K = W_g + W_r$$

Where, W_r is the work done on the raindrop by the resistive force.

$$\therefore \quad W_r = \Delta K - W_g$$

$$= 3.6 \text{ J} - 20 \text{ J}$$

$$= -14.4 \text{ J}$$

31. A man weighing 80 kgf carries a stone of weight 20 kgf to the top of the building 30 m high. Calculate the work done by him. Given $g = 9.8$ ms^{-2}. **[Diksha]**

Ans. Here weight of man = 80 kgf

Weight of stone = 20 kgf

Force applied to carry the total weight up

$$F = 80 + 20$$

$$= 100 \text{ kgf}$$

$$= 100 \times 9.8$$

$$= 980 \text{ N}$$

Height through which the weight is carried

$$s = 30 \text{ m}$$

Work done, $W = Fs = 980 \times 30 = 29{,}400$ J

32. A body is being raised to a height h from the surface of earth, what is the sign of work done by:

(A) Applied force.

(B) Gravitational force.

Ans. Work done, $W = Fs \cos\theta$

(A) The applied force and the displacement are in the same direction, *i.e.*, $\theta = 90°$.

Hence work done is positive.

(B) The gravitational force and the displacement are along with the opposite directions *i.e.*, $\theta = 180°$.

Hence, the work done is negative.

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

33. Two bodies having unequal masses which are moving in the same direction with equal kinetic energy. The two bodies are brought to rest by applying a force of the same magnitude. How would the distance moved by them before coming to rest compare? [NCERT Exemplar]

Ans. Let the two bodies be of masses M_1 and M_2 and moving with speeds u_1 and u_2 respectively. Since they have equal kinetic energy.

$$\frac{1}{2}M_1u_1^2 = \frac{1}{2}M_2u_2^2$$

or $$\frac{u_1^2}{u_2^2} = \frac{M_1}{M_2} \quad \text{...(i)}$$

When the same retarding force F is applied on the two bodies, suppose that they come to rest by covering distances s_1 and s_2 respectively. If a_1 and a_2 are the retardation respectively, then

$$M_1 a_1 = M_2 a_2$$

Or

$$\frac{a_1}{a_2} = \frac{M_1}{M_2} \quad \text{– (ii)}$$

Now, $v^2 - u^2 = 2as$

For the first day,

$$(0)^2 - u_1^2 = 2(-a_1)s_1$$

Or $$u_1^2 = 2a_1s_1$$

Similarly, for the first body,

$$u_2^2 = 2a_2s_2$$

$$\therefore \quad \frac{u_1^2}{u_2^2} = \frac{a_1s_1}{a_2s_2}$$

Using the equation (i) and (ii) the equation (iii) given, $s_1 = s_2$

i.e., the two bodies will come to rest within the same distance.

34. An object having a mass 20 kg initially at rest is subjected to a force of 20 N. What is the kinetic energy acquired by the body at the end of 5 s? [Diksha]

Ans. Mass of object $m = 10$ kg and the force acting upon it is F = 20 N.

Therefore, the acceleration produced in the body is

$$a = \frac{F}{m} = \frac{20\text{ N}}{10\text{ kg}} = 2\text{ ms}^{-2}$$

If v be the velocity of an object, initially at rest, acquired at the end of 15 s, then

$$v = u + at$$
$$= 0 + (2\text{ ms}^{-2} \times 15\text{ s})$$
$$= 30\text{ ms}^{-1}$$

$\therefore$ The kinetic energy of the object at the end of 15 sec is,

$$\text{K.E} = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 10\text{ kg} \times (30\text{ ms}^{-1})^2$$
$$\text{KE} = 4500\text{ J}$$

35. Consider a one dimensional motion of a particle with total energy E. These are four regions A, B, C, D in which the potential energy (v), kinetic energy (K) and total energy (E) is given as below:

(A) Region A: $v > E$ (B) Region B: $v < E$

(C) Region C: $K > E$ (D) Region D: $v > k$

State with reason in each case whether a particle can be found in the given regions or not.

Ans. (A) Region A: $v > E$

The particle cannot be found in region A, as it will lead to a negative value of its kinetic energy.

(B) Region B: $v < E$

The particle can be found in region B, as it will lead to a non-zero value of its kinetic energy.

(C) Region C: $k > E$

The particle can be found in region C. It will lead to a negative value of its potential energy, which is possible.

(D) Region D: $v > E$

The particle can be found in region D, as potential energy can be greater than its kinetic energy.

NUMERICAL Type Questions

36. How high must a body be lifted to gain an amount of P.E. equal to the K.E. it has when moving at speed 20 ms^{-1}? (The value of acceleration due to gravity at a place is 9.8 ms^{-2}) [Delhi Gov. QB 2022] (2m)

Ans. As per conservation of energy.

$$mhg = \frac{1}{2}mv^2$$

i.e., $$9.8\,h = \frac{1}{2}\times 20\times 20$$

$$h = \frac{200}{9.8} = 20.4 \text{ m.}$$

37. A raindrop of mass 2 g falling from a height of 1 km hits the ground with a speed of 40 ms^{-1}. Calculate:

(A) Loss in potential energy of the drop.

(B) Gain in kinetic energy of the drop. (2m)

Ans. (A) The loss of potential energy of the drop.

$$\Delta u = mgh$$

Here, $m = 2g = 2\times 10^{-3}$ kg

$h = 1$ km $= 10^3$m

$g = 10$ ms^{-2}

$\therefore$ $$\Delta u = 2\times 10^{-3}\times 10\times 10^3$$

$$\Delta u = 20 \text{ J}$$

(B) The gain in kinetic energy of the drop

$$\Delta k = \frac{1}{2}mv^2$$

Here, $m = 1$ g $= 2\times 10^{-3}$ kg

$v = 50$ ms^{-1}

$\therefore$ $$\Delta k = \frac{1}{2}\times 2\times 10^{-3}\times (50)^2$$

$$\Delta k = 2.5 \text{ J}$$

38. 20 J work is required to stretch a spring through 0.1 m. Find the force constant of the spring. If the spring is further stretched through 0.1 m. Calculate work done. [Delhi Gov. QB 2022](2m)

Ans. Potential Energy of spring when stretched through a distance 0.1 m.

$$U = \text{W.D.} = \frac{1}{2}Kx^2 = 20 \text{ J}$$

or $K = 4000$ N/m

When spring is further stretched through 0.1 m, then Potential Energy will be:

$$U' = \frac{1}{2}k\,(0.2)^2 = 80 \text{ J}$$

$\therefore$ $$\text{W.D.} = U' - U$$

$$= 80 - 20 = 60 \text{ J}$$

39. A uniform chain of length 2 m is kept on a table, such that a length of 60 m hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain and the table? (2m)

Ans. Here, mass of chain = 4 kg

Length of chain = 2 m

Length of chain hanging freely = 60 cm = 0.6 m

$\therefore$ Mass of the hanging chain

$$m = \frac{4}{2}\times 0.6 = 1.2 \text{ kg}$$

To pull the entire chain on the table, the center of gravity of the hanging out to be raised to the table i.e., hanging part to be raised through a distance

$$h = \frac{0.6}{2} = 0.3 \text{ m}$$

$\therefore$ Work done, $W = mgh$

$$= 1.2\times 10\times 0.3 = 3.6 \text{ J}$$

40. A single conservative force acting on a particle within a system various as $F = (-Ax^2 + Bx^3)\hat{i}$, where A and B are constants, F is in newtons, x is in meters.

(A) Calculate the potential energy function $u(x)$ associated with this force for the system, taking $u = 0$ at $x = 0$.

(B) Find the change in potential energy.

(C) Calculate the change in kinetic energy of the system as the particle moves from $x = 2.00$ m to $x = 3.00$ m (3m)

Ans. (A) The Potential Energy function is calculated by integrating the force, and applying the given condition that, $u(0) = 0$

$$u(x) = \int F dx$$

$$u(x) = -\int(-Ax^2 + bx^3)dx$$

$$u(x) = -\frac{A}{3}x^3 + \frac{B}{4}x^4 + c$$

$$u(0) = 0 = -\frac{A}{3}(0)^3 + \frac{B}{4}(0)^4 + c$$

$$c = 0$$

$$u(x) = -\frac{A}{3}(x)^3 + \frac{B}{4}(x)^4$$

(B) The change in potential energy of this particle from $x = 2$ to $x = 3$ can be calculated simply as:

$\Delta u = u(3) - u(2)$
$= (-9A + 20.25\ B) - \left(\frac{-8A}{3} + 4B\right)$

$\Delta u = (-5A + 16.25\ B)$ J

(C) Because the force is conservative, it must obey conservation of energy without an external input of energy or the influence of a non-conservative dissipative force like air resistance. The total energy, of the particle u changes in some way, then the kinetic energy T must change in the opposite way.

$$E = T + u$$
$$\Delta E = \Delta T + \Delta u$$
$$\Delta E = 0 \text{ (construct)}$$
$$0 = \Delta T + \Delta u$$
$$\Delta T = -\Delta u = -(-5A + 16.25B)$$
$$\Delta T = (5A - 16.25\ B) \text{ J}$$

41. **A ball at rest is dropped from a height of 12 m. It loses 25% of its kinetic energy in striking the ground, find the height to which it bounces. How do you account for the loss in kinetic energy? [Delhi Gov. QB 2022](3m)**

Ans. The kinetic energy on reaching the ground will be equal to the initial potential energy, *i.e.*, KE = $mg(12)$. Since the remaining kinetic energy (after loss) will be used to rise the block to height h,

$$mg(12)(1 - 0.25) = mgh$$

Hence, $h = 9$ m

42. **A stone weighing 1 kg is whirled in a vertical circle at the end of a rope of length 0.5 m. Find the tension in the string at:**

(A) lowest position

(B) midway when the string is horizontal

(C) topmost position to just complete the circle.

[Given: Radius of circle = r = 0.5 m, mass of the body = m = 1 kg, g = 9.8 m/s^2.] (5m)

Ans. (A) Lower most point

$$T_L = \frac{m}{r}(u^2 + gr)$$
$$= \frac{m}{r}(5gr + gr) = 6\ mg$$
$$= 6 \times 1 \times 9.8$$
$$= 58.8 \text{ N}$$

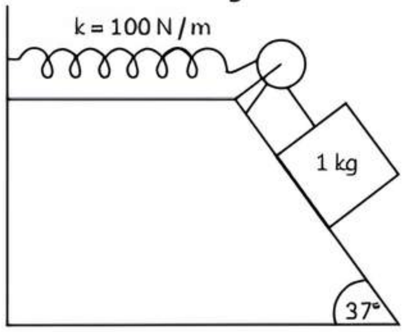
(B) When string is horizontal

$$T_M = \frac{m}{r}(u^2 - 2gr)$$
$$= \frac{m}{r}(5gr - 2gr) = 3\ mg$$
$$= 3 \times 1 \times 9.8$$
$$= 29.4 \text{ N}$$

(c) Top most point

$$T_H = \frac{m}{r}(u^2 - 5gr)$$
$$= \frac{m}{r}(5gr - 5gr) = 0 \text{ N}$$

43. **A 1 kg block situated on a rough incline is connected to a spring with spring constant 100 Nm^{-1} as shown in figure.**



The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the inclined. Assume that the spring has negligible mass and the pulley is frictionless. (5m)

Ans. From figure:

$$R = mg \cos\theta$$
$$F = \mu R = \mu\, mg \cos\theta$$

Net force on the block, down the incline,

$$= mg \sin\theta - F$$
$$= mg \sin\theta - \mu\, mg \cos\theta$$
$$= mg (\sin\theta - \mu \cos\theta)$$

Here, distance moved, x = 10 cm = 0.1 m

In equilibrium,

Work done = potential energy of stretched spring

$$mg(\sin\theta - \mu\cos\theta)\, x = \frac{1}{2}kx^2$$
$$kx = 2mg(\sin\theta - \mu\cos\theta)$$
$$100 \times 0.1 \text{ m} = 21 \text{ kg} \times 10 \text{ ms}^{-1} \sin 51^\circ \mu \cos 37^\circ)$$
$$10 = 20\,(0.601 - \mu \times 0.798)$$
$$\frac{10}{20} = 0.601 - \mu \times 0.798$$
$$0.5 = 0.601 - \mu \times 0.798$$
$$-0.798\mu = 0.5 - 0.601$$
$$\mu = \frac{-0.101}{-0.798} = \frac{101}{798}$$
$$\mu = 0.126$$

POWER AND CONSERVATION OF LINEAR MOMENTUM

2

TOPIC 1

POWER

Power is defined as the rate of doing work. If an agent does work W in time t, then its power is given by,

$$P = \frac{W}{t}$$

Power is a scalar quantity, because it is the ratio of two scalar quantities work (W) and time (t).

Dimensions of Power

$$[P] = \frac{[W]}{[t]} = \frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$$

Units of Power: SI unit of power is watt (W). The power of an agent is one watt if it works at the rate of 1 joule per second.

The bigger units of power are kilowatt (kW) and horsepower (hp).

$$1 \text{ kW} = 1000 \text{ W}$$

$$1 \text{ hp} = 747 \text{ W}$$

Instantaneous Power

The power of an agent may not be constant during a time interval. It is defined as the limiting value of the average. Power as the time interval approaches zero.

If ΔW work is done in a small-time interval Δt, then the instantaneous power is given by;

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

Important

➥ *Relation between kwh and Joule.*

1 kWh = 1 kW × 1 hr = 1000 W × 1 h

= 1000 Js^{-1} × 3600 s

1 kWh = 3.6 × 10^6 J

Conservative and Non-Conservative Forces

Conservative force

A force is said to be conservative if the work is done by or against the force in moving a body, depends only on the initial and final positions of the body and not on the nature of path followed between the initial and the final positions.

This means, work done by one against a conservative force in moving a body over any path between fixed initial and final positions will be same.

For example: Gravitational force is a conservative force.

Non-conservative force

A force is said to be non-conservative if the work is done by or against the force in moving a body from one position to another, depends on the path followed between two positions. Frictional forces are non-conservative forces.

Important

➥ *Work done by a non-conservative force adds or removes mechanical energy from a system.*

Example 2.1: How much mass is converted into energy per day in Tarapur nuclear power plant operated at 10^7 kW?

Ans. Power, $P = 10^7 \text{ kW} = 10^{10} \text{ W} = 10^{10} Js^{-1}$

Time, t = 1day = 24 × 60 × 60 s

Energy produced per day,

$$E = Pt = 10^{10} \times 24 \times 60 \times 60 = 864 \times 10^{12} J$$

As $E = mc^2$

$$m = \frac{E}{c^2} = \frac{864 \times 10^{12}}{(3 \times 10^8)^2}$$

$$= 9.6 \times 10^{-3} \text{ kg}$$

$$= - 9.6 \text{ g}$$

Example 2.2: Consider an object of mass 2 kg moved by an external force 20 N in a surface having coefficient of kinetic friction 0.9 to a distance 10 m. What is the work done by the external force and kinetic friction? Comment on the result. (Assume g = 10 ms^{-2})

Ans. Given, m = 2 kg, d = 10 m

F_{ext} = 20 N, μ_k = 0.9

When an object is in motion on the horizontal surface, it experiences two forces.

(1) External force, F_{ext} = 20 N

(2) Kinetic friction

$$F_k = \mu_c mg$$

$$= 0.9 \times (2) \times 10 = 18 \text{ N}$$

Work done by external force W_{ext}

$$= F_{ext} \times d$$
$$= 20 \text{ N} \times 10 \text{ m}$$
$$= 200 \text{ J}$$

Work done by the force of kinetic friction.

$$W_f = F_k d$$
$$= (-18) \times 10$$
$$= -180 \text{ J}$$

Here, the -ve sign implies that the force of kinetic friction is opposite to the direction of displacement.

Total work done on object

$$W_{total} = W_{ext} + W_k$$
$$= 200 \text{ J} - 180 \text{ J}$$
$$= 20 \text{ J}$$

Since, the friction is a non-conservative force. Out of 200 J given by the external force, the 180 J is lost and it cannot be recovered.

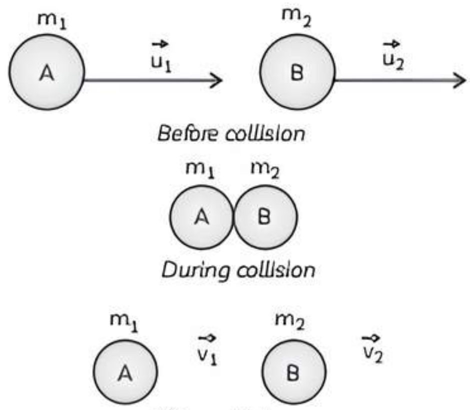
TOPIC 2

COLLISION

Collision between two bodies (or pastilles) is said to occur when they either physically strike against each other, or if the path of the motion of one is influenced by the other.

Example: collision between two billiard balls, carrom coins or automobiles are examples of collision in daily life.

When two bodies collide, they exert interacting forces on each other, which are equal and opposite at all instant during collision. As soon as the collision is over, the interacting forces become zero.



Elastic Collision

Those collisions, in which both momentum and kinetic energy of the system are conserved, are called elastic collisions.

The collision between two bodies and subatomic particles are elastic in nature.

Characteristics:

(1) The momentum is conserved.

(2) The total energy is conserved.

(3) The K.E. is conserved.

(4) M.E. is not converted into any other forms (sound, heat, light) of energy.

(5) Forces involved during the interaction are of conservative nature.

Elastic Collision in One Dimension

The collision, in which both the momentum and kinetic energy are conserved and the colliding bodies continue to move along the same straight line after the collision, it is called an elastic collision in one dimension.

Consider two perfectly elastic bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively.

If $u_1 > u_2$, then the bodies A and B collide head on. Let $\vec{v}_1$ and $\vec{v}_2$ be the new velocities (after collision) of A and B respectively. Since, the collision is heat-on, all velocities are along the same straight line.

By the law of conservation of linear momentum, we have,

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

Since all velocities are in the same direction, we can write the last equation in terms of the magnitude of $\vec{u}_1, \vec{u}_2, \vec{v}_1$ and $\vec{v}_2$. Thus,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \text{...(i)}$$

Since the collision is elastic, the kinetic energy of the bodies is also conserved in the collision. That is,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_1^2$$

$$m_1(u_1^2 + v_1^2) = m_2(v_2^2 + u_2^2) \quad \text{...(ii)}$$

Dividing equation (ii) by equation (i), we have

$$\frac{u_1^2 + v_1^2}{u_1 + v_1} = \frac{v_2^2 + u_2^2}{v_2 + u_2}$$
$$u_1 + v_1 = v_2 + u_2$$
$$u_1 - u_2 = v_2 - v_1 \quad \text{...(iii)}$$

$u_1 - u_2$ is the relative velocity with which the bodies A and B approach each other before collision, and $v_2 - v_1$ is the relative velocity with which the bodies separate from each other after the collision.

Thus, in an elastic one-dimensional collision, the relative velocity of 'approach' after collision is equal to the relative velocity of 'separation' after collision.

We can now find out the final velocity v_1 and v_2 of the bodies A and B in terms of their masses m_1 and m_2 and initial velocity u_1 and u_2 from equation (iii), we have,

$$v_2 = u_1 - u_2 + v_1$$

Put the value of v_2 in equation (i), we have

$$m_1 (u_1 - v_1) = m_2 (u_1 + v_1 - 2u_2)$$

$$-v_1 (m_1 + m_2) = u_1 (m_2 - m_1) - 2m_2 u_2$$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2 \quad \text{...(iv)}$$

This is the expression for the velocity v_1 of the body after the collision.

Similarly, putting $(v_1 = v_2 + u_2 - u_1)$ from equation (iii) in equation (i), we have,

$$m_1 (2u_1 - v_2 - u_2) = m_2 (v_2 - u_2)$$

$$-v_2 (m_1 + m_2) = - u_2 (m_2 - m_1) - 2m_1 u_1$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 \quad m_1}{m_1 + m_2}\right)u_2 \quad \text{...(v)}$$

This is the expression for the velocity v_2 of the body B after the collision. It can also be written from the expression for v_1 (equation iv) by replacing m_1 by m_2 and u_1 by u_2.

Special cases:

(1) If the masses of the colliding bodies are equal

i.e., $m_1 = m_2 = m$ (say).

then from equation (iv) and (v), we have

$$v_1 = u_2 \text{ and } v_2 = u_1$$

i.e., in a one dimensional elastic collision of two bodies of 'equal' masses, the bodies mainly exchange their velocities after the collision.

(2) If the target body (B) is initially at rest.

i.e., $u_2 = 0$,

then from equation (iv) and (v),

we have

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 \quad \text{... (vi)}$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 \quad \text{... (vii)}$$

(i) The target body (B) which is at rest is much more massive than the colliding body (A).

that is $m_2 >> m_1$;

Then, ignoring m_1 in equation (vi) and (vii), we have

$$v_1 = u_1 \text{ and } v_2 = 2u_1$$

i.e., when a light moving body suffers a one-dimensional elastic collision with a much heavier body at rest, the lighter body rebounds with its velocity, while the heavier body remains practically at rest.

(ii) The target body (B), which is at rest, is much lighter than the colliding body (A) *i.e.*, $m_2 << m_1$;

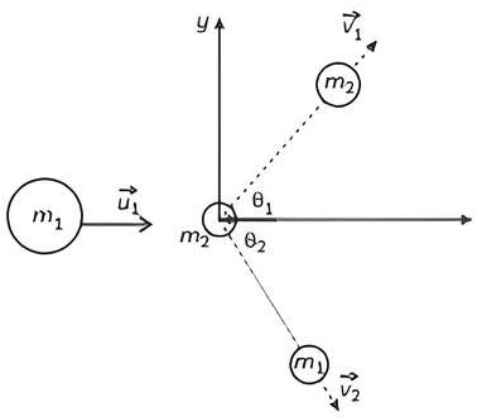
Then ignoring m_2 in equation (vi) and (vii), we have,

$$v_1 = u_1 \text{ and } v_2 = 2u_1$$

i.e., when a heavy moving body collides with a much lighter body at rest, the heavier body continues moving with almost the same velocity, while the lighter body starts with approximately twice the velocity of the heavier body.

Elastic Collision in Two Dimensions

Suppose an elastic of mass m_1 moving with velocity $\vec{u}_1$ along a straight line, say x-axis, collides with another elastic body of mass m_2, initially at rest. After collision, the bodies m_1 and m_2 move with velocities $\vec{v}_1$ and $\vec{v}_2$ in the xy-plane angles θ_1 and θ_2 respectively with the x-axis.



By the law of conservation of linear momentum, We have,

$$m_1\vec{u}_1 = m_1\vec{v}_1 + m_2\vec{v}_2$$

Let us take the scalar components of $\vec{u}_1, \vec{v}_1$ and $\vec{v}_2$ along the x and y directions and apply the law of conservation of linear momentum separately the x and y components of motion, then we have,

$$m_1 u_1 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \quad \text{...(i)}$$

$$0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2 \quad \text{...(ii)}$$

Since the collision in elastic, the K.E. is also conserved *i.e.*,

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1u_1^2 = m_1v_1^2 + m_2v_2^2 \quad \text{... (iii)}$$

In equation (i), (ii) and (iii), we may assume that m_1, m_2 and u_1 are known, but the motion after the collision involves four unknown quantities v_1, v_2, θ_1 and θ_2 which all cannot be determined from three equations. At least one of them should be given.

Important

➥ *Those collisions, for which angle (θ) is nearly zero so that is practically, 90°, are called glancing collisions.*

Inelastic Collision

Those collisions, in which the momentum of the system is conserved, but the kinetic energy is not conserved, are called inelastic collisions.

In this collision, most of the kinetic energy of the bodies is converted into other forms such as heat, light, sound, etc.

Inelastic Collision in One Dimension

Let there be an inelastic head – on collision between two bodies of masses m_1 and m_2, initially moving with velocities $\vec{u}_1$ and $\vec{u}_2$ respectively. After colliding, the bodies stick together and move with the same velocity $\vec{v}_1$. By the law of conservation of momentum.

We have,

$$m_1\vec{u}_1 + m_2\vec{u}_2 = (m_1 m_2)\vec{v}$$

Since, all velocities are along the same line, and we can write the above equation as,

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\,v \qquad \text{–(i)}$$

∴ If the masses of the body and their initial velocities be known, then the final velocity can be calculated.

The K.E. of the system before collision is,

$$K_1 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

and the K.E. of the system after collision is,

$$K_2 = \frac{1}{2}(m_1 + m_2)v^2$$

If m_2 be initially at rest ($u_2 = 0$) then the ratio of kinetic energies after collision and before collision is,

$$\frac{K_2}{K_1} = \frac{\frac{1}{2}(m_1+m_2)v^2}{\frac{1}{2}m_1u_1^2}$$

In this condition ($u_2 = 0$), substituting the value of v from equation (i),

we get

$$\frac{K_2}{K_1} = \frac{(m_1+m_2)m_1u_1^2}{(m_1+m_2)^2m_1u_1^2} = \frac{m_1}{m_1+m_2}$$

We see that K_2 and K_1. Thus, the inelastic collision results in a loss of kinetic energy.

In practice, there is no collision which is perfectly elastic or perfectly inelastic.

Example 2.3: Case Based:

A ballistic pendulum is a device for measuring a bullets' momentum, from which, it is possible to calculate the velocity and kinetic energy. A collision between two objects must either be elastic or inelastic. In an elastic collision both the momentum and the kinetic energy of the system are conserved. On the other hand, in an inelastic collision momentum is conserved, and the two objects stick together after the collision. In the case of a ballistic pendulum, the collision is inelastic because the bullet is embedded in the block.

(A) The length of a ballistic pendulum is 1 m and the mass of its block is 1.9 kg. A bullet of mass 0.1 kg strikes the block of ballistic pendulum in horizontal direction with a velocity 100 ms^{-1} and got embedded in the blank after collision the combined mass (block and bullet) swings away from the lowest point. What is the tension in the string when it makes an angle 60° with vertical? (g = 10ms^{-2})

(B) The length of the ballistic pendulum is 1 m and the mass of its block is 0.98 kg. A bullet of mass 20 g strikes the block along horizontal direction and gets embedded in the block. If (block + bullet) completes a vertical circle of radius 1 m, the striking velocity of the bullet is:

(a) 280 m/s

(b) 313 m/s

(c) 420 m/s

(d) 490 m/s

(C) Assertion (A): If the bob of a simple pendulum is kept in a horizontal electric field, its period of oscillation will remain same.

Reason (R): If Bob is charged and kept in a horizontal electric field, then the time period will be decreased.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

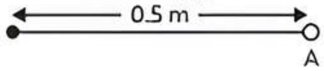
(c) A is true but R is false.

(d) A is false and R is also false.

(D) A simple pendulum is oscillating with an angular amplitude 60°. If the mass of the bob is 50 g, the tension in the string at mean position is: (g = 10 m/s²)

(a) 0.5 N (b) 1 N
(c) 1.5 N (d) 2 N

(E) The bob of a simple pendulum is held in the horizontal position A as shown in fig. Calculate the speed of the bob of the pendulum at the lower position B when released. The length of the pendulum is 0.5 m and it is assumed that there is no loss of energy.



Ans. (A) Momentum conservation

$$0.1 \times 100 = (1.9 + 0.1)v_0$$
$$v_0 = 5 \text{ ms}^{-1}$$
$$H = L(1 - \cos 60°) = \frac{1}{2} = \frac{1}{2}\text{ m}$$

Energy conservation.

$$\frac{1}{2}(2)(5)^2 = \frac{1}{2} \times 2 \times v^2 + 2 \times 10 \times \frac{1}{2}$$
$$= 25 - 10 = v^2$$
$$v = \sqrt{15}\text{ m/s}$$

By force balance.

$$T - mg\cos 60° = m\frac{v^2}{L}$$
$$T = 2 \times 10 \times \frac{1}{2} + 2 \times \frac{15}{1}$$
$$= 40\text{ N}$$

(B) *(b) 313 m/s*

Explanation: Using conservation of momentum.

Initial momentum of bullet = Final momentum of bullet and pendulum

$$0.02 \times v_i = (0.02 + 0.98)v_f$$

So, $v_i = 50 \times v_f$

Kinetic energy after collision = potential energy

$$\frac{1}{2} \times 1 \times v_f^2 = 1 \times g \times 2$$
$$v_f = 2\sqrt{g}$$

Thus, $v_i = 50 \times 2\sqrt{g}$
$$= 313\text{ m/s}$$

(C) *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: When the bob is placed in an electric field, the time period of the simple pendulum will remain the same as the bob is not charged. If a simple pendulum having a charged bob is placed in a horizontal electric field. Then the period will be decreased because there will be an increase in restoring force.

(D) *(b) 1 N*

Explanation: H = R (1 – cos 60°)

$$H = \frac{R}{2}$$

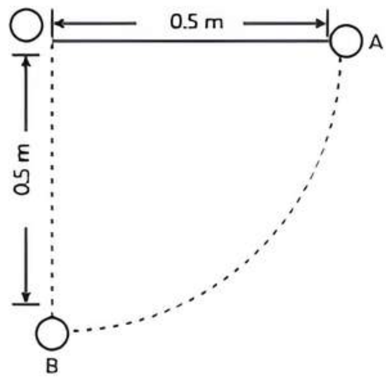
Apply work - Energy theorem.

$$mg\frac{R}{2} = \frac{1}{2}mv^2$$
$$v = \sqrt{Rg}$$

At mean position.

$$T - mg = \frac{mv^2}{R}$$
$$T - 2mg = 2 \times 0.05 \times 10$$
$$= 1\text{ N}$$

(E) At point A, the energy of the pendulum is entirely P.E. at point B, the energy of a pendulum is entirely K.E. It means as the bob of the pendulum lowers from A to B, P.E. is converted into K.E. Thus, at B, K.E = P.E.



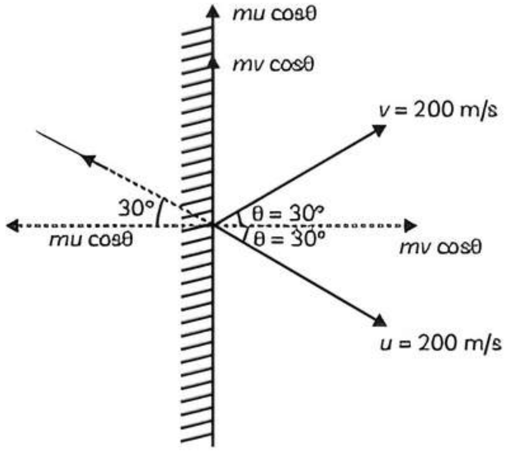
or $$\frac{1}{2}mv^2 = mgh$$
$$v^2 = 2gh = 2 \times 9.8 \times 0.5 = 9.8$$
$$v = \sqrt{9.8} = 3.13\text{ m/s}$$

Example 2.4: A molecule in a gas container hits a horizontal wall with speed 200 ms⁻¹ and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Ans. Let us consider the mass of the molecule be m and that of wall be M. The wall remains at rest due to its large mass. Resolving momentum of the molecule along x-axis and y-axis, we get

The x-component of momentum of molecule

$$= mu\cos\theta$$
$$= -m\,200\cos 30°$$
$$= -100\sqrt{3}\,m$$



y-component of the molecule

$$= mu \sin\theta$$
$$= m \times 200 \times \sin 30^\circ$$
$$= 100 \text{ m}$$

Before collision: x-component of total momentum (wall + molecule)

$$= 0 + (-100\sqrt{3} \text{ m})$$
$$= -100\sqrt{3} \text{ m}$$

y-component of momentum (wall + molecule)

$$= 0 + 100 \text{ m}$$
$$= 100 \text{ m}$$

After collision: x-component of the momentum (wall + molecule)

$$= 0 + \text{m } 200 \cos 30^\circ$$
$$= 100\sqrt{3} \text{ m}$$

and y-component $= 0 + \text{m } 100 \sin 30^\circ$

$$= 100 \text{ m}$$

We find that momentum of the (molecule + wall) system is conserved. The wall has a recoil momentum such that momentum of the wall + momentum of outgoing molecule equals the momentum of the incoming molecule.

Initial kinetic energy $\left(\frac{1}{2}mu^2\right)$ is the same as final K.E. $\left(\frac{1}{2}mv^2\right)$ of the molecule as $u = v = 200$ m/s *i.e.*, Thus, the collision is elastic collision.

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. An engine pumps water continuously through a hose pipe. Water passes through the pipe and leaves it with a velocity of 2 m/s. The mass per unit length of the water in the pipe is 100 kgm^{-1}. What is the power of the engine?

(a) 100 W **(b) 200 W**

(c) 400 W **(d) 800 W**

[NCERT Exemplar]

Ans. *(c) 400 W*

Explanation: The rates of which kinetic energy is imparted to the water, *i.e.*,

Power of the engine,

$$P = \frac{1}{2} mv^3$$

Here, $m = 100$ kgm^{-1}, $v = 2$ ms^{-1}

$$\therefore \quad P = \frac{1}{2} \times 100 \times 2^3$$
$$= 400 \text{ W.}$$

2. A shell, in its flight, explodes into four unequal parts. Which of the following is conserved?

(a) Momentum **(b) Kinetic energy**

(c) Potential energy **(d) Both (b) and (c)**

[Diksha]

Ans. *(a) Momentum*

Explanation: Momentum of an object is equal to the mass of the object times the velocity of the object. Kinetic energy is the energy of an object or a system's particles in motion. Potential energy is the stored energy in any object or system by virtue of its position or arrangement of parts.

Related Theory

➥ *The relation $E = mc^2$ leads to unification of the two laws - law of conservation of mass and law of conservation of energy into one law - called the law of conservation of mass-energy.*

Conversion used:

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{J}$$
$$1 \text{ MeV} = 10^6 \text{ eV} = 10.6 \times 10^{-13} \text{J}$$
$$1 \text{ amu} = 931 \text{ MeV}$$

3. The potential energy of a spring when stretched through a distance x is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance x?

(a) 20 J **(b) 40 J**

(c) 30 J **(d) 45 J**

Ans. *(c) 30 J*

Explanation: P.E. of the spring when stretched through a distance x,

$$u = \frac{1}{2} kx^2 = 10 \text{ J}$$

When x becomes $2x$, the potential energy will be

$$u' = \frac{1}{2}k(2x)^2$$

$$= 4 \times \frac{1}{2}kx^2$$

$$= 4 \times 10 = 40 \text{ J}$$

$\therefore$ Work done $= u' - u$

$$= 40 - 10 = 30 \text{ J}$$

Related Theory

The notion of potential energy applies to only those forces where the work done against its force gets stored up as the energy by the virtue of position or configuration of the body when external constraints are removed. This energy appears as kinetic energy, when the position or configuration of the body gets changed under the action of external constraints.

4. A railway engine of mass 12,000 kg is moving at a constant speed of 5 ms^{-1} up an inclined plane of 15°. Calculate the power of the engine. Given, g = 9.8 ms^{-2}.

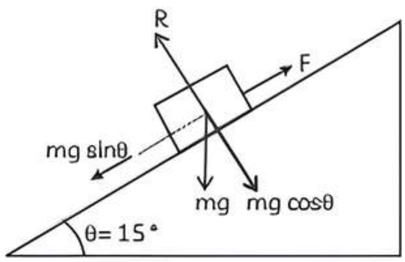
(a) 152.19 W (b) 140.29 W

(c) 129.29 W (d) 119.59 W

[NCERT Exemplar]

Ans. *(a) 152.19 W*

Explanation: The various forces acting on the engine, as shown in figure.



Since, the engine is moving upward, so it is doing work against the force = $mg \sin\theta$.

$\therefore$ The minimum force required is given by,

$$F = mg \sin\theta$$

$$F = 12000 \times 9.8 \times \sin 15°$$

$$F = 30.437 \text{ N}$$

$$v = 5 \text{ ms}^{-1}$$

$$\text{Power} = F \times v$$

$$P = 30.437 \times 5$$

$$P = 152.186 \text{ W}$$

$$P = 152.19 \text{ W.}$$

5. A shell is fixed from a canon with a velocity v (m/s) at an angle θ with the horizontal direction. At the highest point in its path, it explodes into two pieces of equal mass. One of the pieces retraces its path to the canon, and the speed (in m/s) of the other piece immediately after the explosion is:

(a) $3v\cos\theta$ (b) $2v\cos\theta$

(c) $\frac{3}{2}v\cos\theta$ (d) $\frac{\sqrt{3}}{2}v\cos\theta$

Ans. *(a) $3v\cos\theta$*

Explanation: By the principle of conservation of momentum,

$$mv\cos\theta = -\frac{mv\cos\theta}{2} + \frac{m}{2}v'$$

i.e., $$\left(1+\frac{1}{2}\right)mv\cos\theta = \frac{m}{2}v'$$

i.e., $$v' = 3v\cos\theta$$

6. A person does some work of 1000 J in 2s. His power will be

(a) 1000 W (b) 25 W

(c) 500 W (d) 50 W [Diksha]

Ans. *(c) 500 W*

Explanation: Given that:

Work (W) = 1000 J

Time (t) = 2 s

Power (P) = ?

We know that $P = \frac{W}{t}$

$$\text{Power, } P = \frac{1000}{2} \text{ J/s}$$

$$\text{Power, } P = 500 \text{ W}$$

7. A stone of mass 0.012 kg moving with a velocity of 70 m/s gets embedded in a freely suspended wooden block of 0.4 kg. What is the speed acquired by the block?

(a) 0.94 kg ms^{-1} (b) 0.84 kg ms^{-1}

(c) 0.62 kg ms^{-1} (d) 0.58 kg ms^{-1}

Ans. *(b) 0.84 kg ms^{-1}*

Explanation: Initial momentum of stone = mv

Here,

$$m = 0.012 \text{ kg}$$

$$v = 70 \text{ ms}$$

As we know that,

$$p = mv$$

$$p = 0.012 \text{ kg} \times 70 \text{ ms}^{-1}$$

$$p = 0.84 \text{ kg ms}^{-1}$$

8. An electric heater of rating 1000 W is used for 5 hours per day for 20 days. What is the electric energy utilised?

(a) 100 kWh (b) 200 kWh

(c) 120 kWh (d) 500 kWh

Ans. *(a) 100 kWh*

Explanation: The power of the electric heater is 1000 W, the time period is

$20 \times 5 = 100$ hr

Electrical energy = Power × Time

$= 1 \times 100$

$= 100$ kWh

Related Theory

➥ *Kilowatt hour (kWh) or Board of Trade (BOT) is the commercial unit of electrical energy. Relation is also given, between kWh and Joule: by 1 kWh = 3.6×10^6 J*

9. During inelastic collision between two bodies, which of the following quantities always remain conserved.

(a) Total kinetic energy

(b) Total potential energy

(c) Total linear momentum

(d) None of the above [Delhi Gov. SQP 2022]

Ans. *(c) Total linear momentum*

Explanation: Such a collision between two objects, in which some energy is wasted, is known as an inelastic collision. Momentum is conserved in the event of an inelastic collision, but kinetic energy is not. In daily life, most collisions are of an inelastic kind.

Assertion-Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these question from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false

10. Assertion (A): According to the principle of conservation of energy, all heat can be converted into mechanical energy.

Reason (R): Due to various losses, it is impossible to convert all heat into mechanical work.

Ans. *(d) A is false and R is also false*

Explanation: According to the law of conservation of energy, energy can neither be created nor it can be destroyed. Thus, it is physically possible to convert heat into mechanical work. But due to various energy losses, this cannot be achieved partially ever.

Caution

➥ *Students should know that the principle of conservation of energy cannot be proved mathematically, but is an empirical principle. The deductions made on the basis of this principle are found to be true.*

11. Assertion (A): In an elastic collision of two bodies, the momentum and energy of each body is conserved.

Reason (R): If two bodies stick to each other, after colliding the collision is said to be perfectly elastic. [Diksha]

Ans. *(d) A is false and R is also false*

Explanation: In an elastic collision, both the momentum and kinetic energy remains conserved. But this rule is not for individual bodies, but for the system of bodies before and after the collision. While collision in which there occurs some loss of kinetic energy is called inelastic collision. Collision in daily life and generally inelastic. The collision is said to be perfectly inelastic, if two bodies strike each other.

Caution

➥ *Students should know that those collisions, in which both momentum and kinetic energy of the system are conserved, are called elastic collisions.*

12. Assertion (A): In an elastic collision between the two bodies, the relative speed of the bodies after collisions is equal to the relative speed before the collision.

Reason (R): In a elastic collision the linear momentum of the system is conserved.

[Delhi Gov. QB 2022]

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: In Collisions:

(1) Momentum is preserved in both elastic and inelastic collisions.

(2) Kinetic energy is preserved in elastic collisions as well.

Point 1 leads to Reason, which states that in an elastic collision, the system's linear momentum is conserved.

Point 2 leads to the Assertion, in an Elastic Collision between two bodies, the relative speed of the bodies after the collision is equal to the relative speed before the collision.

13. Assertion (A): A bullet is fired from a rifle. If the rifle recoils freely, the kinetic energy of the rifle is more than that of the bullet.

Reason (R): In the case of the rifle bullet system, the law of conservation of momentum violates.

Ans. *(d) A is false and R is also false.*

Explanation: Law of conservation of linear momentum is correct when no external force acts. When a bullet is fired from a rifle then both should pass equal momentum but different kinetic energy

$$E = \frac{P^2}{2m}$$

∴ Kinetic energy of the rifle is less than that of the bullet because $E \propto \frac{1}{m}$

Caution

↪ *Students must know that if a force is conservative, it is possible to assign a numerical value for the potential at any point.*

14. **Assertion (A):** **Total energy of the freely falling body is constant at each point.**

Reason (R): **Kinetic energy of freely falling body is minimum, when it reaches the ground.**

Ans. *(c) A is true but R is false.*

Explanation: According to the law of conservation of energy, energy of a freely falling body remains conserved. Kinetic energy is maximum when body reaches at ground.

15. **Assertion (A):** **The work done during a round trip is always zero.**

Reason (R): **No force is required to move a body in its round trip.**

Ans. *(d) A is false and R is also false.*

Explanation: In a round trip, work done is zero only when the force is conservative in nature. Force is always required to move a body in a conservative, non-conservative field.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

16. *In billiards if the cue ball strikes a stationary billiard ball straight on, then the cue ball will stop moving after the collision. It will have transferred all of its kinetic energy to the other ball, which will move forward with the same velocity that the cue ball had before the collision. Collisions can only be elastic if the masses are equal. The masses of billiard balls are the same, which can make some collisions close to elastic.*

(A) Two spheres A and B of masses m_1 and m_2 respectively collide. A is at rest initially and B is moving with velocity v along x-axis. After collision B has a velocity $\frac{v}{2}$ in a direction perpendicular to the original direction. The mass A moves after collision in the direction:

(a) $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ to the x-axis

(b) $\theta = \tan^{-1}\left(-\frac{1}{2}\right)$ to the y-axis

(c) Same as that of B

(d) Opposite to that of B

(B) Two bodies with kinetic energies in the ratio of 4 : 1 are moving with equal linear momentum. The ratio of their masses is:

(a) 4 : 1 (b) 1 : 1

(c) 1 : 2 (d) 1 : 4

(C) Two balls of masses m each are moving at right angle to each other with velocities 6 m/s and 8 m/s respectively. If collision between them is perfectly inelastic, the velocity of combined mass is:

(a) 15 m/s (b) 10 m/s

(c) 5 m/s (d) 2.5 m/s

(D) Body A of mass 4 m moving with speed u collides with another body B of mass 2 m at rest. The collision is head on and elastic in nature. After the collision the fraction of energy lost by the colliding body A is:

(a) $\frac{1}{9}$ (b) $\frac{8}{9}$

(c) $\frac{4}{9}$ (d) $\frac{5}{9}$

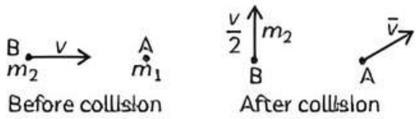
(E) A moving block having mass m, collides with another stationary block having mass $4m$. The lighter block comes to rest after collision. When the initial velocity of the lighter block is v, then the value of coefficient of restitution (e) will be:

(a) 0.5 (b) 0.25

(c) 0.4 (d) 0.8

Ans. (A) *(a) $\theta = \tan^{-1}\left(\frac{1}{2}\right)$ to the x-axis*

Explanation:



$$m_2 v\hat{i} + 0 = -m_2\frac{v}{2}\hat{j} + m_1\bar{v}$$

Using momentum conservation.

$$m_1\bar{v} = m_2 v\hat{i} + m_2\frac{v}{2}\hat{j}$$

$$\theta = \tan^{-1}\left(\frac{v}{2v}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

Angle is from x-axis.

(B) *(d) 1 : 4*

Explanation:

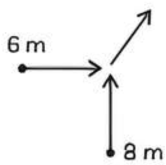
$$\frac{K_1}{K_2} = \frac{P_1^2 2m_2}{2m_1 P_2^2} \qquad (P_1 = P_2 \text{ given})$$

$$\Rightarrow \quad \frac{K_1}{K_2} = \frac{m_2}{m_1} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \frac{1}{4}$$

(C) *(c) 5 m/s*

Explanation: Using momentum conservation.



$$m\sqrt{6^2+8^2} = 2mv'$$

$$v' = 5 \text{ m/s}$$

(D) *(b)* $\frac{8}{9}$

Explanation: Fractional loss of KE of colliding body.

$$\frac{\Delta KE}{KE} = \frac{4(m_1 m_2)}{(m_1+m_2)^2}$$

$$= \frac{4(4m)2m}{(4m+2m)^2}$$

$$= \frac{32\,m^2}{36\,m^2} = \frac{8}{9}$$

(E) *(b) 0.25*

Explanation: According to law of conservation of linear momentum.

$Mv + 4m \times 0 = 4m' + 0$

$$v' = \frac{v}{4}$$

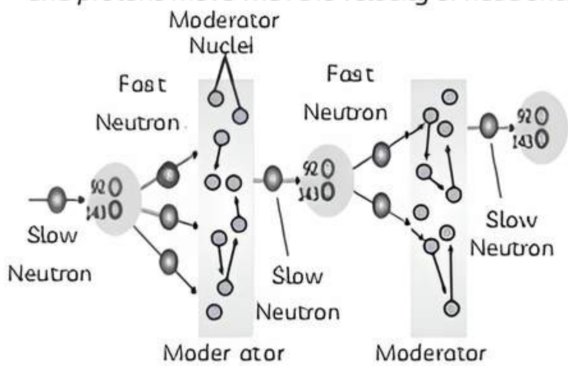
$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of appoach}}$$

$$= \frac{\frac{v}{4}}{v}$$

$$e = \frac{1}{4}$$

$$= 0.25$$

17. *Hydrogenic material is used as a moderator in nuclear reactors to slow down the neutrons. As we know that when two bodies of same mass undergo elastic collision, their velocities are interchanged after collision. Water and heavy water are hydrogenic materials containing protons having approximately the same mass as that of a neutron. When fast moving neutrons collide with protons, the neutrons come to rest and protons move with the velocity of neutrons.*



(A) **A molecule in a gas container hits a horizontal wall with speed and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?**

(B) **If two bodies stick together after collision will the collision be elastic or inelastic?**

(C) **If two objects collide and one is initially at rest (i) is it possible for both to be at rest after collision? (ii) is it possible for any one to be at rest after collision?**

Ans. (A) Yes. Collision is elastic.

The momentum of the gas molecule remains conserved whether the collision is elastic or inelastic.

The gas molecule moves with a velocity of 200 m/s and strikes the stationary wall of the container, rebounding with the same speed.

It shows that the rebound velocity of the wall remains zero. Hence, the total kinetic energy of the molecule remains conserved during the collision. The given collision is an example of an elastic collision.

(B) Inelastic collision

(C) (i) No, because momentum will not be conserved in that case

(ii) Yes, when masses of two objects are equal and collision is perfectly elastic.

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

18. Calculate the speed of the bob of a simple pendulum at its mean position if it is able to rise to a vertical height of 10 cm. Take $g = 9.8\ m/s^2$. [NCERT Exemplar]

Ans. By conservation of energy.

K.E. of the bob at the mean position

= P.E. of the bob at the highest position

$$\frac{1}{2}mv^2 = mgh$$

$$\therefore \quad v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.10}$$

$$v = \sqrt{1.96}$$

$$v = 1.4\ m/s$$

19. A ball bounces to 80% of its original height. What fraction of its mechanical energy is lost in each bounce? Where does this energy go?

Ans. Suppose the ball is dropped from height h.

Initial P.E. = mgh

P.E. after first bounce

= $mg \times$ 80% of h

= $0.80\ mgh$

P.E. lost in each bounce = $0.20\ mgh$

Fraction of P.E. lost in each bounce

$$= \frac{0.20mgh}{mgh} = 0.20$$

This energy is lost in the form of heat, sound, etc.

20. A falls under from a height of 10 m with an initial downward velocity u. It collides with the ground, loses 50% of its energy in collision and then rises back to the same height. Find the initial velocity u. [Diksha]

Ans. If m is the mass of the car, then its total initial energy at height h.

$$= \frac{1}{2}mu^2 + mgh$$

Energy after collision

$$= 50\% \text{ of } (\frac{1}{2}mu^2 + mgh)$$

$$= \frac{1}{2}(\frac{1}{2}mu + mgh)$$

As the car rebounds to height, so

$$\frac{1}{2}\left(\frac{1}{2}mu^2 + mgh\right) = mgh$$

$$\frac{1}{4}mu^2 = \frac{1}{2}mgh$$

$$u = \sqrt{2gh}$$

$$= \sqrt{2 \times 9.8 \times 10}$$

$$= 14\ m/s$$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

21. A man weighing 50 kgf carries a load of 10 kgf to the top of a building in 4 minutes. The work done by him is 6×10^4 J. If he carries the same load in 8 minutes, what will be the work he's done? [NCERT Exemplar]

Ans. Work done = mgh ...(i)

$m = 50\ kg + 10\ kg = 60\ kg$

$g = 10\ m/s^2$

$h = ?$

Substituting values, we get

$6 \times 10^4 = 60 \times 10 \times h$ ...(ii)

$6 \times 10000 = 6 \times 10 \times 10\ h$

$h = 100$ m

Putting the value of h in equation (ii),

$6 \times 10^4 = 60 \times 10 \times 10 \times 100$

$60000 = 60000$

$6 \times 10^4 J = 6 \times 10^4 J$

L.H.S = R.H.S

Let us assume that height was given instead of work done. ($h = 100$ m)

Then,

Work done = mgh

$W = 60 \times 10 \times 100$ J

$W = 60000$ J

$W = 6 \times 10^4$ J

22. **The pulse frequency of a person is 90 beats min^{-1} and at each beat the heart discharges 70 ml of blood against a pressure of 80 mm of mercury. Find the power of the heart, if the density of mercury is 143.6 × 10^3 kg m^{-3}. Take, g = 10 m/s^2.**

Ans. Here, Pulse frequency = 90 beats min^{-1}

Volume of blood discharged by the heart at each beat

$$= 70 \text{ ml} = 70 \times 10^{-6} \text{ m}^3$$

∴ Volume of blood discharged by the heart per minute (during 90 beats)

$$V = 70 \times 10^{-6} \times 90$$
$$= 63 \times 10^{-4} \text{ m}^3$$

Pressure at which the blood is discharged,

P = 80 mm of mercury

$$= 80 \times 10^{-3} \text{ m of mercury}$$
$$= 80 \times 10^{-3} \times 13.6 \times 10^3 \times 10$$
$$= 10{,}880 \text{ Nm}^{-2}$$

Now, work done by the heart per minute

$$W = \text{force} \times \text{distance}$$
$$= (\text{pressure} \times \text{area}) \times \text{distance}$$
$$= PV = 10880 \times 63 \times 10^{-4}$$
$$= 68.54 \text{ J}$$

Hence, power of the heart

$$P = \frac{W}{t} = \frac{68.54}{60} = 1.14 \text{ W}$$

23. **A tube-well pumps out 2400 kg of water per minute. The water is coming out with a velocity of 3.0 m/s. Find the power of the pump. How much work is done it if the pump runs for 10 hrs?**

Ans. The mass of water pumped out per second is,

$$m = \frac{2400 \text{ kg}}{60 \text{ s}} = 40 \text{ kg s}^{-1}$$

The kinetic energy of water pumped out per second is,

$$\frac{1}{2}mv^2 = \frac{1}{2} \times 40 \times (3.0)^2 = 180 \text{ Js}^{-1}$$

This is equal to the work done per second, that is the power of the pump.

Thus, $P = 180 \text{ Js}^{-1} = 180 \text{ W}$

Work done in 10 hrs (= 36000 s) is

$$W = P \times t = 180 \text{ W} + 36000 \text{ s}$$
$$= 6.48 \times 10^6 \text{ J}$$

SHORT ANSWER Type-II Questions (SA-II)

[**3** marks]

24. **Using the expression for power and K.E. of rotational motion, derive the relation τ = Iα.** **[Delhi Gov. SQP 2022]**

Ans. We have the power of rotational motion,

$$P = \tau\omega \quad ...(i)$$

and the K.E. of rotational motion,

$$K.E. = \frac{1}{2}I\omega^2 \quad ...(ii)$$

Now, the power of rotational motion is equal to time rate of work done during the rotational motion. Since, the work done is stored in the form of kinetic energy.

$$P = \frac{d}{dt}(\text{K.E. or rotational motion})$$

Using the equations (i) and (ii), we have

$$\tau\omega = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right) = \frac{1}{2}I\frac{d}{dt}(\omega^2)$$
$$= \frac{1}{2}I(2\omega)\frac{d\omega}{dt}$$
$$= \frac{1}{2}I(2\omega)\alpha = I\omega\alpha$$

or, $\tau = I\alpha$

25. **A child sits stationary at one end of a long trolley moving uniformly with a speed v on a smooth horizontal hour. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?** **[Diksha]**

Ans. The child is running on a trolley moving with velocity v. However, the running of the child will produce no effect on the velocity of the centre of mass of the trolley. This is because the force due to the boy's motion is purely internal. Internal forces produce no effect on the motion of the bodies on which they act. Since, no external force is involved in the boy-trolley system, the boy's motion will produce no change in the velocity of the centre of mass of the trolley.

26. **Distinguish between a head on and an oblique collision.**

Ans. If the colliding objects move along the straight line joining their centers, the collision is said to be a head-on collision.

If the colliding objects do not move along the straight line joining their centers, the collision is said to be an oblique collision.

27. **Two bodies A and B having masses m_A and m_B respectively have equal K.E. If P_A and P_B are their respective momentum, then prove that the ratio of momentum is equal to the square root of the ratio of respective masses.** **[Delhi Gov. SQP 2022]**

Ans. Let V_A and V_B the velocities of A and B respectively.

$$\frac{1}{2}m_A V_A^2 = \frac{1}{2}m_B V_B^2 \quad \text{...(i)}$$

or, $m_A V_A^2 = m_B V_B^2$

or, $(m_A V_A)V_A = (m_B V_B)V_B$

or, $P_A V_A = P_B V_B$

or, $$\frac{P_A}{P_B} = \frac{V_A}{V_B} \quad \text{...(ii)}$$

From equation (i), we get

$$\frac{V_A^2}{V_B^2} = \frac{m_B}{m_A}$$

or, $$\frac{V_A}{V_B} = \sqrt{\frac{m_B}{m_A}}$$

From equation (ii), we get

$$\frac{P_A}{P_B} = \sqrt{\frac{m_A}{m_B}}$$

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

28. A bus having mass 3000 kg is lifted up to a distance of 30 m by a crane in 1 minute. A second crane does the same job in 2 min. Do the cranes consume the same or different amounts of fuel? What is the power supplied by each crane? Neglect power dissipation against friction (g = 9.8 m/s^2). [Diksha]

Ans. In order to lift a body against gravity through a height, a force equal to the weight of the body is applied vertically upwards, that is, in the direction of the displacement. Hence, the work done (force × displacement) in the direction of force is, (According to the Work Energy Theorem)

$$W = mg \times h$$

Here, $W = 2000 \times 9.8 \times 30$

$= 5.8 \times 10^5$ J

Since, both cranes do the same amount of work, they will consume the same amount of energy, *i.e.*, the same amount of fuel.

The two cranes take different times, 1 min = 60 sec and 2 min = 120 sec in doing the same work. Hence, their powers are different. The power of the first crane is,

$$P_1 = \frac{W}{t_1} = \frac{5.88 \times 10^5 \text{ J}}{60 s} = 9800 \text{ W}$$

and power of second crane,

$$P_2 = \frac{W}{t_2} = \frac{5.88 \times 10^5 \text{ J}}{120 s} = 4900 \text{ W}$$

29. A body of mass 5 kg moves along the x-axis with a velocity of 2 m/s. A second body of mass 10 kg moves along the y-axis with a velocity of $\sqrt{3}$ ms^{-1}. They collide at the origin and stick together.

Calculate:

(A) The final velocity of the combined mass after collision.

(B) The amount of heat liberated in the collision. [NCERT Exemplar]

Ans. (A) Suppose, the two masses m_1 and m_2 have velocities v_1 and v_2 respectively and the velocity of the combined mass ($m_1 + m_2$) is v, making angle θ with x-axis.

By momentum-conservation along the x-axis, we have

$$m_1 v_1 = (m_1 + m_2)\, v \sin\theta$$

Putting the values

$$5 \text{ kg} \times 2 \text{ ms}^{-1} = (5+10) \text{ kg } (v \cos\theta)$$

$$10 \text{ kg} \times \sqrt{3} \text{ ms}^{-1} = (5 + 10) \text{ kg } (v \cos\theta)$$

Solving, we get

$$v = \frac{4}{3} \text{ ms}^{-1}$$

And

$$\tan\theta = \sqrt{3}$$

$$\theta = 60°$$

(B) The initial kinetic energy of the system is:

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}\times 5 \times (2)^2 + \frac{1}{2}\times 10 \times (\sqrt{3})^2$$

$$= 10 + 15 = 25 \text{ J}$$

The final kinetic energy is:

$$\frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}\times 15 \times (4/3)^2 = \frac{40}{3} \text{ J}$$

The lost kinetic energy appears as heat:

$$\therefore \text{Heat liberated} = 25 - \frac{40}{3} = \frac{35}{3} \text{ J}$$

30. A railway carriage of mass, 15000 kg moving with a speed of 64 km/h strikes a stationary carriage of the same mass. The two carriages get coupled and move together. What is their common speed? Is the collision elastic?

Ans. Let, m be the mass of each carriage and u the speed of the moving carriage. Thus, the total momentum of the carriage before collision is

$$mu = 15000 \text{ kg} \times 64 \text{ km/hr}$$

$$= 9,60,000 \text{ kg km/hr}$$

The total momentum of the carriage after the collision is,

$$2\,mv = 30000\, v \text{ kg}$$

Where, v is the common of the coupled carriages

By conservation of momentum, we have

$$30000\, v \text{ kg} = 960000 \text{ kg km/hr}$$

$$v = 32 \text{ km/hr}$$

or

Which is half the speed of the moving carriage.

$\left(v = \frac{v}{2}\right)$

The initial kinetic energy is $\frac{1}{2}mu^2$.

while the final kinetic energy is,

$$\frac{1}{2}(2m)v^2 = \frac{1}{2}(2m)(4/2)^2 = \frac{1}{2}mu^2$$

The final kinetic energy is half the initial kinetic energy, that is the kinetic energy is conserved Hence, the collision is inelastic.

NUMERICAL Type Questions

31. A man pulls a lawn roller through a distance of 20 m with a force of 20 kg weight. If he applies the force at an angle of 60° with the ground, calculate the power developed if he takes 1 min in doing so.

[Delhi Gov. QB 2022] (2m)

Ans. As we know that,

$$P = \frac{W}{t} = \frac{Fs\cos\theta}{t} = 32.66 \text{ W}$$

32. A railway carriage of mass 9000 kg moving with a speed of 36 kmh^{-1} collides with a stationary carriage of same mass. After the collision, the carriages get coupled and move together. What is their common speed after collision? What type of collision is this?

[Delhi Gov. QB 2022](3m)

Ans. Given, $m_1 = 90000$ kg, $u_1 = 36$ km/h $= 10$ m/s

$m_2 = 90000$ kg, $u_2 = 0$, $v = v_1 = v_2 = ?$

By conservation of momentum:

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$\therefore \quad v = 5$ m/s

Total K.E. before collision $= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

$= 45000$ J

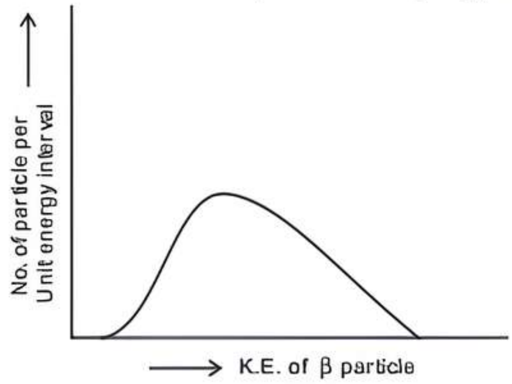
Total K.E. after collision $= \frac{1}{2}(m_1 + m_2)v^2$

$= 225000$ J

As total K.E. after collision < total K.E. before collision

$\therefore$ Collision is inelastic.

33. Consider the decay of a free neutron at rest: $n > p + e^-$. Show that the two body decay of this type must necessary given an electron of fixed energy and therefore, cannot account for the observed continuous energy distribution in the β-decay of a neutron or a nucleus. **[NCERT Exemplar](5m)**



Ans. Let the masses of the electron and proton be m and M respectively.

Let u and v be the velocities of electrons and protons, respectively.

Using law of conservation of momentum,

Momentum of electron + momentum of proton = momentum of neutron

$\therefore \quad mu + mv = 0$

$\Rightarrow \quad v = -\frac{m}{M}$

Clearly, the electron and the proton move in opposite directions. If mass Δm has been converted into energy in the reaction, then,

$$\frac{1}{2}mu^2 + \frac{1}{2}mv^2 = \Delta m \times c^2$$

$$\frac{1}{2}mv^2 + \frac{1}{2}M\left[-\frac{m}{M}\right]^2 u^2 = \Delta mc^2$$

$$\frac{1}{2}mu^2\left[1 + \frac{m}{M}\right] = \Delta mc^2$$

$$u^2 = \frac{2M\Delta mc^2}{m(M+m)}$$

Thus, it is proved that the value of u^2 is fixed. Since, all the quantities in the right-hand side are constant. It establishes that the emitted electron must have a fixed energy, and thus we cannot account for the continuous energy distribution in the β-decay of a neutron.